8100 Lecture- "J"

November 14, 2019

Suppose firms have the following technology:

$$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

Prices of inputs are:

$$w_1 = 4, w_2 = 1$$

Let's now assume all firms have x_1 fixed at $x_2 = 1$.

$c(q) = 4q^2 + 1$

$mc(q) = \frac{\partial(4q^2+1)}{\partial q} = 8q$

The supply function of the firm is: $mc(q) = p$.

$$q = \frac{1}{8}p$$

There are J firms, and so market supply is:

$$Q_s = \frac{J}{8}p$$

Suppose the demand is:

$$Q_d = \frac{294}{p}$$

Equilibrium is such that $Q_d = Q_s$.

$$\frac{J}{8}p = \frac{294}{p}$$

Equilibrium price is:

$$p = \frac{28\sqrt{3}}{\sqrt{J}}$$

The equilibrium quantity is:

$$\frac{J}{8}\frac{28\sqrt{3}}{\sqrt{J}} = \frac{7\sqrt{3}\sqrt{J}}{2}$$

Individual firm quantity is:

$$\frac{1}{8}\frac{28\sqrt{3}}{\sqrt{J}} = \frac{7\sqrt{3}}{2\sqrt{J}}$$

Individual firm profit:

$$\frac{7\sqrt{3}}{2\sqrt{J}}\frac{28\sqrt{3}}{\sqrt{J}} - \left(4\left(\frac{7\sqrt{3}}{2\sqrt{J}}\right)^2 + 1\right) = \frac{147}{J} - 1$$

2.	72.5
3.	48.
5.	28.4
10.	13.7
100.	0.47
1000.	−0.85

Let's see how good our assumption is:

$$\pi'(q) = p'(q)\,q + p - c'(q)$$

We assume that $p'(q) = 0$. Is this "valid"?

$$\frac{\partial p(q)}{\partial q} = \frac{\partial\left(\frac{294}{q}\right)}{\partial q} = -\frac{294}{q^2}$$

Let's plug in for q.

$$-\frac{294}{\left(\frac{7\sqrt{3}\sqrt{J}}{2}\right)^2} = -\frac{8}{J}$$

2.	4.
3.	2.67
5.	1.6
10.	0.8
100.	0.08
1000.	0.01

Let's assume there is a monopolist. $J = 1$. This firm's profit function is:

$$\frac{294}{q} q - \left(4q^2 + 1\right) \quad = \quad 293 - 4q^2$$

There is no maximum, however, profit is decreasing in q. The monopolist wants to set $q = 0$. On the other hand, if they assume price is fixed:

$$q = \frac{7.0\sqrt{3}}{2} = 6.06218$$

Profit under the assumption that price does not depend on q:

$$146$$

Now suppose we return to J firms, but they take serious that price is determined by the inverse demand function. q_i is the quantity of firm i. $i \in \{1, 2, ..., J\}$. Profit is given by $q_{-i} = \left(\sum_{i=1}^{J} q_i\right) - q_i$:

$$\pi_i\left(q_i, q_{-i}\right) = \frac{294}{q_i + q_{-i}} q_i - \left(4q_i^2 + 1\right)$$

The stationary condition for firm i is:

$$\frac{\partial\left(\frac{294}{q_i+q_{-i}} q_i - \left(4q_i^2 + 1\right)\right)}{\partial q_i} \quad = \quad \left(\frac{294}{q_{-i} + q_i} - \frac{294 q_i}{\left(q_{-i} + q_i\right)^2}\right) - 8q_i = MR - MC$$

The optimal level of q_i is given by:

$$\left(\frac{294}{q_{-i} + q_i} - \frac{294 q_i}{\left(q_{-i} + q_i\right)^2}\right) - 8q_i = 0$$

Let's assume $q_i = q_j \forall i, j$. Call q, the quantity for an individual firm.

$$\left(\frac{294}{(J)\, q} - \frac{294 q}{(J * q)^2}\right) - 8q = 0$$

$$q = \frac{7\sqrt{3}\sqrt{J-1}}{2J}$$

Market quantity is:

$$Q_s = \frac{7\sqrt{3}\sqrt{J-1}}{2}$$

Equilibrium price:

$$p = \frac{294}{\frac{7\sqrt{3}\sqrt{J-1}}{2}} = \frac{28\sqrt{3}}{\sqrt{J-1}}$$

Individual firm profit is:

$$\frac{28\sqrt{3}}{\sqrt{J-1}}\frac{7\sqrt{3}\sqrt{J-1}}{2J} - \left(4\left(\frac{7\sqrt{3}\sqrt{J-1}}{2J}\right)^2 + 1\right) \quad = \quad \frac{294}{J} - \frac{147(J-1)}{J^2} - 1$$

Recall that under the assumption of perfect competition, firm profit was:
Individual firm profit:

$$\frac{7\sqrt{3}}{2\sqrt{J}}\frac{28\sqrt{3}}{\sqrt{J}} - \left(4\left(\frac{7\sqrt{3}}{2\sqrt{J}}\right)^2 + 1\right) \quad = \quad \frac{147}{J} - 1$$

Tables in terms of J. Cournot model first.

2.	109.25
3.	64.33
5.	34.28
10.	15.17
100.	0.48
1000.	−0.85

2.	72.5
3.	48.
5.	28.4
10.	13.7
100.	0.47
1000.	−0.85

Let's now compare the market quantities to determine the distortion introduced by the price-taking assumption:

$$Q_{cournot} = \frac{7\sqrt{3}\sqrt{J-1}}{2}$$

$$Q_{comptition} = \frac{7\sqrt{3}\sqrt{J}}{2}$$

$$\frac{Q_{cournot}}{Q_{competition}} = \frac{\sqrt{J-1}}{\sqrt{J}}$$

A table of this in terms of J:

2.	0.71
3.	0.82
5.	0.89
10.	0.95
100.	0.99
1000.	1.