Collusion and Surplus

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1 Monopoly Markup.

A monopolist has profit function:

$$\pi = p(q)\, q - c(q)$$

The First-order condition is:

$$\frac{\partial p(q)}{\partial q} q + p(q) = mc(q)$$

$$p(q) = mc(q) - \frac{\partial p(q)}{\partial q} q$$

Price is equal to Marginal cost, plus some term.

$$p = mc - \frac{1}{\epsilon} p$$

$$p = mc \frac{\epsilon}{1+\epsilon}$$

If demand is elastic, then $\epsilon < -1$ and $\frac{\epsilon}{1+\epsilon} > 1$. In this case, this is a markup over marginal cost pricing.

$$p = mc \frac{\epsilon}{1+\epsilon}$$

In fact, a monopolist will always operate in the elastic portion of the demand curve. If they weren't, then a reduction in quantity would increase their revenue, and also decrease their cost. To see this, note that the derivative of revenue with respect to quantity is negative when demand is elastic.

$$\frac{\partial\left[qp\left(q\right)\right]}{\partial q}=p\left(q\right)+qp'\left(q\right)$$

$$\frac{\partial\left[qp\left(q\right)\right]}{\partial q}=p\left(q\right)+p\left(q\right)\frac{qp'\left(q\right)}{p\left(q\right)}=p\left(q\right)\left[1+\frac{1}{\epsilon}\right]$$

When $p\left(q\right)>0$, this is negative when:

$$\left[1+\frac{1}{\epsilon}\right]<0$$

$$-1<\epsilon<0$$

Note that this has little to do with the structure of the monopolists revenue. It is true more generally that for a function $f\left(x\right)>0$, the product $xf\left(x\right)$ is decreasing in x if and only if the elasticity of f with respect to x is between 0 and -1. This is a nice property of elasticity to remember.

2 Collusion

2.1 Cournot Baseline

Let's look at a very simple cournot model with $p=294-Q$, $c\left(q_i\right)=4q_i$. The profit function of the firm:

$$\pi\left(q_i\right)=\left(294-Q_{-i}-q_i\right)q_i-4q_i$$

Marginal profit is:

$$\frac{\partial\left(\left(294-Q_{-i}-q_i\right)q_i-4q_i\right)}{\partial q_i} \quad = \quad -2q_i-Q_{-i}+290$$

The first-order condition:

$$\frac{290-Q_{-i}}{2}=q_i$$

In a symmetric equilibrium:

$$\frac{290-\left(J-1\right)q}{2}=q$$

Thus, individual q and market quantity Q are:

$$\frac{290}{J+1} = q$$

$$\frac{J}{J+1}290 = Q$$

For instance, when $J = 2$: $q = 96.6667$,$Q = 193.333$, $p = 100.667$, $\pi = 9344.48$

2.2 Collusion of 2 firms.

What if two firms jointly maximize their profit by both setting q_c.

$$\pi(q_c) = (294 - q_c - q_c)\, q_c - 4q_c$$

$$\frac{\partial\left((294 - q_c - q_c)\, q_c - 4q_c\right)}{\partial q_c} = 290 - 4q_c$$

Each firm sets:

$$\frac{290}{4} = q_c$$

Market Quantity:

$$\frac{290}{2} = Q$$

Profit of each firm under collusion:

$$\pi = 10512.5$$

2.3 Collusion of 2 firms with J total.

Think about how you might solve the optimization problem of 2 firms trying to collude in a market with J total firms. When do they have incentive to collude?

3 Measuring Welfare.

3.1 Producer Surplus.

We have seen that producer surplus can be written as $VC(q) - p(q)\, q = q\left[AVC(q) - p(q)\right]$. This can also be written as:

$$\int_0^q \left[MC(z) - p(q)\right] dz = VC(q) - qp(q)$$

3.2 Consumer Surplus.

Producer surplus is technically an approximation of utility gains from the existence of a market measured in monetary terms. It is the area below the inverse demand function and above price.

$$\int_0^q \left[p\left(\zeta\right) - p\left(q\right)\right] d\zeta$$