Welfare.

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1 Measuring Welfare.

1.1 Cournot Competition

Let's look at a simple Cournot model with constant marginal cost of 4. Inverse demand is $p = 294 - Q$. With this we can write the firm's profit function:

$$\pi_i = (294 - Q)\, q_i - (4q_i)$$

$$\pi_i = (294 - (Q_{-i} + q_i))\, q_i - (4q_i)$$

Maximizing this:

$$\frac{\partial\left((294 - (Q_{-i} + q_i))\, q_i - (4q_i)\right)}{\partial q_i} = -2q_i - Q_{-i} + 290$$

To find a symmetric equilibrium, we can set $q_i = q$ for all i.

$$\frac{290 - (J-1)\, q}{2} = q$$

$$q = \frac{290}{J+1}$$

$$Q = 290\frac{J}{J+1}$$

$$p = 294 - 290\frac{J}{J+1}$$

1.2 Consumer Surplus.

$$\int_0^Q \left[(294 - z) - 294 + 290\frac{J}{J+1}\right] dz$$

$$\frac{290\frac{J}{J+1}\left[290\frac{J}{J+1}\right]}{2} = \frac{1}{2}\left(290\frac{J}{J+1}\right)^2$$

Under monopoly, this is 10512.5, while under perfect competition it is 42050. (Four times larger).

1.3 Producer Surplus

Total Producer surplus. Area above supply (marginal cost) and below price.

$$\left(294 - 290\frac{J}{J+1} - 4\right) 290\frac{J}{J+1}$$

$$290^2\frac{J}{(J+1)^2} \quad = \quad \frac{84100J}{(J+1)^2}$$

Notice that this is decreasinging in J.

1.4 Total Surplus

$$290^2\frac{J}{(J+1)^2} + \frac{1}{2}\left(290\frac{J}{J+1}\right)^2 + = 290^2\frac{1}{(J+1)^2}\left[J + \frac{1}{2}J^2\right]$$

$$290^2\frac{1}{(J+1)^2}\left[J + \frac{1}{2}J^2\right] \quad = \quad \frac{84100}{(J+1)^2\left(\frac{J^2}{2} + J\right)}$$

1.5 Deadweight-loss.

This is the difference between total suplus in perfect competition and with J firms:

$$\frac{290^2}{2} - 290^2\frac{1}{(J+1)^2}\left[J + \frac{1}{2}J^2\right] \quad = \quad 42050 - \frac{84100}{(J+1)^2\left(\frac{J^2}{2} + J\right)}$$

2 Optimal Splitting

We have previous assumed that $c = 4q^2 + 1$ for each firm. But if this technology is available to all firms, why can't a monopolist simply buy a new "version" of the same technology. Let's assume this and see what happens.

A monopolist can buy new factory for 1 dollar that produces output q_i according to the cost function $c_i = 4q_i^2$. We ask, how many factories should the monopolist buy? The demand function is $p = 294 - Q$.

First step is to write the cost function as a function of production q and factories J. We know, since there are increasing marginal costs, the cheapest way of producing q with J factories is to produced $\frac{q}{J}$ at each factory.

$$c(q, J) = J\left(4\left(\frac{q}{J}\right)^2\right) + J$$

Let's try to minimize this by choosing J for a fixed q.

$$J\left(4\left(\frac{q}{J}\right)^2\right) + J = \frac{4q^2}{J} + J$$

This is minimized where it is stationary.

$$\frac{\partial\left(\frac{4q^2}{J} + J\right)}{\partial J} = 1 - \frac{4q^2}{J^2}$$

$$J^2 = 4q^2$$

The cost minimizing number of factories is:

$$J = 2q$$

This allows us to write the new cost minimized (over J) cost function for the monopolist.

$$c(q) = 2q\left(4\left(\frac{q}{J}\right)^2\right) + 2q$$

$$c(q) = 4q$$

Since demand is $p = 294 - q$, marginal revenue is $294 - 2q$.The monopolist maximizes profit where marginal cost is equal to marginal revenue.

$$q = 145$$

$$p = 149$$

The number of factories is 290. Per-factory production is $\frac{1}{2}$. What happened? Notice that if we write the average total cost: $\frac{4q_i^2+1}{q_i}$. This is minimized at:

$$\frac{\partial\left(\frac{4q_i^2+1}{q_i}\right)}{\partial q_i} = 8 - \frac{4q_i^2+1}{q_i^2}$$

$$4q_i^2 = 1$$

$$q_i = \frac{1}{2}$$

Further, the avereage cost at $q_i = \frac{1}{2}$ is 4. If the monopolist can buy new factories, it will run them al at the most efficient production level possible.