

Elasticity

Suppose the price of a good changes from 2 to 1. Consumer 1's demand changes from 100 to 50 and consumer 2's changes from 10 to 5. Their behavior in terms of absolute changes in demand $\frac{\Delta x_i}{\Delta p_i}$ is wildly different, but their behavior in terms of percentage terms $\frac{\frac{\Delta x_i}{x_i}}{\frac{\Delta p_i}{p_i}}$ is identical. Elasticity is simply a way of quantifying comparative statics in unit-free percentage terms.

Price Elasticity:

$$\frac{\partial x_i}{\partial p_i} \frac{p_i}{x_i} = \varepsilon_{i,i}$$

Cross Price Elasticity:

$$\frac{\partial x_i}{\partial p_j} \frac{p_j}{x_i} = \varepsilon_{i,j}$$

Income Elasticity:

$$\frac{\partial x_i}{\partial y} \frac{y}{x_i} = \eta_{i,j}$$

Elasticity Relations

While there are few things we can say for sure about the system of consumer demands without further assumptions, one thing we know is that if the consumer has locally-nonsatiated preferences, the budget equation will hold at the optimum.

$$y = \sum_{j \in I} p_j x_j(p, y)$$

From this, we can derive several relationships in the system of demands, by taking the derivative of both sides of this equation.

Taking the derivative with respect to price and manipulating the result yields:

$$-s_i = \sum s_j \varepsilon_{i,j}$$

Taking the derivative with respect to income and manipulating the result yields:

$$1 = \sum_{j \in I} s_j \eta_j$$

An example of using the complementary slackness condition to check corner-solutions

$$u(x) = \log(x_1) + \sqrt{x_2} + x_3$$

$$x_1 + x_2 + x_3 \leq m$$

The Lagrangian function:

$$\log(x) + \sqrt{y} + z - \lambda(x_1 + x_2 + x_3 - m) - \mu_1(-x_1) - \mu_2(-x_2) - \mu_3(-x_3)$$

The first order conditions are:

$$\mu_1 + \frac{1}{x_1} = \lambda$$

$$\mu_2 + \frac{1}{2\sqrt{x_2}} = \lambda$$

$$\mu_3 + 1 = \lambda$$

Suppose only the budget equation binds:

$$x_1 = 1$$

$$\frac{1}{\sqrt{2}} = x_2$$

$$\lambda = 1$$

Since the marginal utility per dollar of x_3 is always 1, if any x_3 is consumed, the marginal utility per dollar for all goods must be 1.