A Simple Market

Profit function for a Cobb Douglas firm is with $\alpha = \frac{1}{4}$ and $w_1 = w_2 = 1$:

$$Max\left(y * p - 2y^2\right)$$

This is concave. FOC is sufficient for a maximum:

$$\frac{\partial\left(y * p - 2y^2\right)}{\partial y} \quad = \quad \frac{p}{4} = y$$

Suppose there are 4 of these firms, market supply is:

$$Y_s = 4\frac{p}{4}$$

Suppose demand for this is:

$$Y_d = \frac{100}{p}$$

$$p = 10$$

$$y = \frac{10}{4}$$

$$\pi = \frac{25}{2}$$

Suppose it costs \$1 to create a firm. Firms will enter if profit is positive. The fixed cost does not affect the optimal level of output, so:

$$Y_s = J\frac{p}{4}$$

In equilibrium:

$$J\frac{p}{4} = \frac{100}{p}$$

$$\left(\frac{400}{J}\right)^{\frac{1}{2}} = p$$

$$y = \frac{\left(\frac{400}{J}\right)^{\frac{1}{2}}}{4} = \frac{5}{\sqrt{J}}$$

$$\pi = \frac{\left(\frac{400}{J}\right)^{\frac{1}{2}}}{4} * \left(\frac{400}{J}\right)^{\frac{1}{2}} - 2\left(\frac{\left(\frac{400}{J}\right)^{\frac{1}{2}}}{4}\right)^2 - 1$$

$$= \frac{1}{8}\left(\frac{400}{J}\right) - 1$$

The extra -1 in the above profit function comes from the fixed cost of entering:

$$Table[\frac{1}{8}\left(\frac{400}{J}\right) - 1, \{J, \{1, 2, 3, 4, 50\}\}]$$

$$50 = J$$

How reasonable is the price taking assumption?

In the previous problem the firms assume profit is:

$$\pi = py - 2y^2$$

And thus:

$$MR = p = \frac{20}{\sqrt{J}}$$

However, in reality it is:

$$MR = p + \frac{\partial p(y)}{\partial y}y$$

$\frac{\partial p(y)}{\partial y}y$ is the indirect effect of increasing production. If production of one firm is increased, then aggregate productn is increased as well. But this will lower what consumers are willing to pay. In fact, they are willing to pay at most $p(Y) = \frac{100}{Y}$ for aggregate output Y. Thus, $\frac{\partial p(y)}{\partial y}y = -\frac{100}{Y^2}y$

$$MR = p + \frac{\partial p(y)}{\partial y}y = \frac{20}{\sqrt{J}} + \left(-\frac{100}{Y^2}\right)y$$

Plugging in the equilibrium values of firm output y and aggregate ouput Y:

$$MR = \frac{20}{\sqrt{J}} + \left(-\frac{100}{\left(5\sqrt{J}\right)^2}\right)\frac{5}{\sqrt{J}} = \frac{20}{\sqrt{J}} - \frac{20}{J^{\frac{3}{2}}}$$

Thus, compared to the assumption of price-take, actual marginal revenue is lower by $\frac{20}{J^{\frac{3}{2}}}$. How far off is the actual marginal revenue from what it is assumed to be under price-taking depending on the number of firms:

$$\begin{pmatrix} 2 & 0.5 \\ 3 & 0.666667 \\ 5 & 0.8 \\ 10 & 0.9 \\ 100 & 0.99 \end{pmatrix}$$

For two firms, we are off by 50% in making the price-taking assumption. But for 100 firms, we are only off by 1%. For a large number of firms, predictions based on price-taking won't be too distorted, but for a small number, they will be way off.

By the way, this was generated with:

$$Table[\{J, \frac{\frac{20}{\sqrt{J}} - \frac{20}{J^{\frac{3}{2}}}}{\frac{20}{\sqrt{J}}}\}, \{J, \{2, 3, 5, 10, 100\}\}]$$