Monopoly

In the last lecture, we saw how far off we can be using the price-taking assumption for a small number of firms. Let's go to the other extreme and assume we have a monopoly. We will use the same production function as in the last lecture:

$$f(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{1}{4}}$$

Prices of inputs are:

$$w_1 = 1, w_2 = 1$$

Cost function:

$$c(y) = 2y^2$$

The inverse demand function is:

$$p(y) = \frac{100}{y}$$

The most consumers will pay for y is $\frac{100}{y}$. Thus the price the monopolist can charge for y units is given by the inverse demand function. The profit function is:

$$\pi(y) = \frac{100}{y} y - 2y^2 = 100 - 2y^2$$

Something weird is going on here. The revenue is 100 regardless of output. Notice that the consumers have constant elasticity of demand.

$$\varepsilon_p = \frac{\partial \left(\frac{100}{p}\right)}{\partial p} \frac{p}{\frac{100}{p}} = -1$$

Reducing output by 1% will allow the monopolist to increase price by 1%, leading to zero change in revenue. However, costs will decrease, thus profit can be increased at any point by decreasing output. In fact, this will be true of any monopolist facing consumer with inelastic or unit-elastic demand. Profit can always be increased by reducing output. Thus *a monopolist will never operate where demand is inelastic or unit elastic.*

In fact, what a monopolist charges can be measured in terms of marginal cost and a "markup" that depends on elasticity of demand:

$$\pi(y) = p(y) y - c(y)$$

The monopolist solves:

$$p + \frac{\partial p}{\partial y} y = MC(y)$$

$$1 + \frac{\partial p}{\partial y}\frac{y}{p} = \frac{MC(y)}{p}$$

$$1 + \frac{1}{\varepsilon} = \frac{MC(y)}{p}$$

$$p = \frac{\varepsilon}{1+\varepsilon}(MC(y))$$

Notice, since a monopolist will only operate where demand is elastic, $\varepsilon < -1$ and:

$$\frac{\varepsilon}{1+\varepsilon} > 1$$

Thus:

$$p > MC$$