A traveler lives in a convex cone. Coordinates in her world are given by points (x_1, x_2)*. She lives at* $(0, 0)$*. There is a particularly nice spot to visit located at* $(2, 2)$*. At the moment, she is able to travel in any straight line including diagonals. Because of this, her travel distance to a point is simply its Euclidean distance* $\sqrt{x_1^2 + x_2^2}$*. Her preferences can be represented by 100 minus the* ***square*** *of her distance to* $(2, 2)$ *and minus the distance she had to travel to get to the point she visits. Thus her utility can be written:*

$$U(x_1, x_2) = 100 - (2 - x_1)^2 - (2 - x_2)^2 - \sqrt{x_1^2 + x_2^2}$$

A) Argue that $x_1 = x_2$ at the optimum.

B) What point does she visit?

C) If the government outlaws traveling on a diagonal so she can only move east-west or north-south, how does her behavior change?

Kevin is a musician. He produces hit tracks using a rented studio space s, creativity-enhancing drinks d, and hourly help from a song-writing assistant a. Kevin's continuous, strictly concave, and strictly monotonic production function is $f(s, d, a)$. Kevin's goal is to minimize the cost of producing at least 4 hit tracks since that is what his label demands. In additional, Kevin has a contract with his song-writing assistant and must use at least 10 hours of help.

Kevin faces a constrained cost minimization. Using a Lagrange multiplier approach, Kevin forms the Lagrangian function.

$$p_s s + p_d d + p_a a + \lambda (4 - f(s, d, a)) + \mu (10 - a)$$

A) Suppose Kevin would use less than 10 hours of his assistant's time if he could. Give an interpretation for the value of μ.

B) Derive an expression for the value of λ at the optimum and give an interpretation of its value assuming $\mu = 0$.

C) Drive an expression for how Kevin's minimized cost of producing four hits changes as the price of drinks changes.

A individual consumes bundles in $\mathbb{R}^2_+$ *and has a preference relation* $\succsim$ *on these bundles that can be represented by the utility function* $U(x_1, x_2) = 2ln\left(x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}\right)$.

- **A)** State what it means for a consumer's preferences to be *homothetic* in terms of their preference relation.
- **B)** Prove this consumer has homothetic preferences.
- **C)** Find the Marshallian Demand for each good.
- **D)** Confirm Roy's Identity.
- **E)** What is the elasticity of substitution for the consumer? Interpret this number.
- **F)** How does this consumer's demand compare to a consumer with the utility $U(x_1, x_2) = \left(x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}\right)^2$.