

## Exam 2.

### harmonic ii.

Consider following production function:

$$f(x_1, x_2) = \left( \frac{1}{x_1} + \frac{1}{x_2} \right)^{-1}$$

A. Write down a function that gives  $x_2$  as a function of  $x_1$  and  $y$  and represents the isoquant for production level  $y$ .

B. Prove this function is convex in  $x_1$  for positive  $x_1$  and  $x_2$

C. Is the function  $f$  quasi-concave? How do you know?

D. Is the function  $f$  homogeneous? If so, of what degree?

E. Is the function  $f$  concave? How do you know?

### dj max leisure : mc min work.

As you may know, rappers have the following production function for producing *hits* where  $x_1$  is time and  $x_2$  is talent:

$$f(x_1, x_2) = 40 \left( \frac{1}{x_1} + \frac{1}{x_2} \right)^{-1}$$

Talent is measured in terms of Kanyes. For Kanye  $x_2 = 1$ . For a rapper half as talented as Kanye,  $x_2 = \frac{1}{2}$ . Time is measured in lifetimes spent in the studio. Thus,  $x_1 \leq 1$ .

A. What is the elasticity of substitution between time and talent? Interpret this in a way that a non-economist would understand.

Talent is not fixed. Rappers have some initial talent and can increase talent by spending time with Kanye. Talent increases by one-quarter of the proportion of a lifetime spent with Kanye. For instance, if a rapper with initial talent 0 spends half of his/her lifetime with Kanye, his/her talent would be  $\frac{1}{8}$ . Since he/she only has one lifetime, if the rest of that lifetime was spent in the studio, this rapper would have  $x_1 = \frac{1}{2}$  and  $x_2 = \frac{1}{8}$ . Any time not spent in the studio or with Kanye is spent as *leisure time*.

B. Suppose a rapper with initial talent  $\frac{1}{3}$  wants to produce 10 hits and otherwise wants to maximize leisure time. If he/she spends no time with Kanye, how much time does he/she need to spend in the studio? If he/she spends  $\frac{1}{3}$  of a lifetime with Kanye, how much time does he/she need to spend in the studio? Which option affords more leisure time?

C. What is the optimal amount of time for a rapper with initial talent  $\frac{1}{3}$  to spend with Kanye to produce 10 hits and maximize leisure time?

D. A rapper produced 10 hits and was maximizing her leisure time, we don't know how much time she spent in the studio, but we know she did not spend any time with Kanye. What is the least amount of talent she must have?

**some gambles.**

A consumer has a utility function with the expected utility property.

Lottery A yields \$10 with probability  $\frac{1}{2}$  and \$0 with probability  $\frac{1}{2}$ .

Lottery B yields \$4 with probability 1.

Lottery C yields \$10 with probability  $\frac{1}{4}$ , \$4 with probability  $\frac{1}{2}$ , and \$0 with probability  $\frac{1}{4}$ .

A. The consumer prefers lottery A to lottery B. Can we infer anything about the consumer's preference between A and C? If so, what can we infer and why?

B. Suppose this consumer has a utility for wealth of  $v(w) = \frac{1}{2}w^{\frac{1}{2}}$ . Is this consumer risk loving, risk averse, or risk neutral?

C. The consumer has initial wealth of  $w_0$ . Write down an equation for the certainty equivalent of lottery A in terms of  $w_0$ .

D. (this is for fun) Would you rather have a lottery that pays \$10 with probability  $\frac{1}{2}$  or \$10 for sure if you answered part d of the previous problem correctly?