Raw Class Notes for 3012- Spring 2022

These notes are unedited versions of the notes we typed in class. For more polished notes, please see "Class Notes" on my webpage.

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1 Class 1- 1/19/2022

X is the **feasible set**.

This a set of bundles. x.

We want the feasible set to be all the relevant bundles for a model.

For example, if we are modeling the choice of ice cream bowls. The bowl: "one scoop of vanilla and one of chocolate" is a single bundles in the set of feasible bundles.

A bundle will normally (in this class) consist of two goods.

For example the two goods might be chocolate ice cream and vanilla ice cream.

a bundle is now an amount of each good.

Let's say good 1 is chocolate and good 2 is vanilla.

$x = (1, 1)$ is the bundle "one scoop of chocolate and one scoop of vanilla.

$(0, 2)$ is two scoops of vanilla.

$(5, 1.8)$ five scoops of chocolate and 1.8 scoops of vanilla.

We could add strawberry to the model. now we have 3 goods.

$(1, 1, 1)$ one scoop of each flavor.

Two goods is "enough" to learn about trade-offs so we work with 2.

Let's go back to chocolate and vanilla ice cream.

Let's define the feasible set for this model. We want the feasible set to be all bowls of ice cream with a positive (non-negative) real number of scoops of each flavor.

$$X = \mathbb{R}^2_+$$

$(0, 2), (1000, 5), (1000000, 29)$ all in the feasible set.

$(-1, 2)$ is not.

$$x = (1, 1) = (x_1, x_2)$$

x_1 is the amount of good 1. and x_2 is the amount of good 2.

Budget set is the set of bundles actually available to a particular consumer. B is the **budget set**.

Budget set might be all the bowls of ice cream with no more than two total scoops. The budget set is always a subset of the feasible set.

$$B \subseteq X$$

Let's write formally the set of all bowls of ice cream with no more than two total scoops.

$$B = \left\{x | x \in \mathbb{R}^2_+ \,\&\, x_1 + x_2 \leq 2\right\}$$

$$B = \left\{x|x \in \mathbb{R}^2\,\&x_1 \geq 0\&x_2 \geq 0\&\,x_1 + x_2 \leq 2\right\}$$

We could a weird budget set:

$$B = \left\{x|x \in \mathbb{R}^2_+\& \left(\sqrt{x_1^2 + x_2^2} \leq 1\right)\right\}$$

This is the the set of all bundles less than distance one from the origin. It is a circle. This is technically possible in our framework.

Normally we think of budgets as coming from income m and prices p_1 and p_2. *Competitive budgets.* The price of any bundle is:

$$p_1x_1 + p_2x_2$$

scoops of ice cream cost \$2 each $p_1 = 2$ and $p_2 = 2$. The cost of the bundle $(2, 1)$ is:

$$(2 * 2) + (2 * 1) = 6$$

Suppose I have m dollars. What can I afford.

$$p_1x_1 + p_2x_2 \leq m$$

This is the formal version of a *competitive budget.*

$$B = \left\{x|x \in \mathbb{R}^2_+\&p_1x_1 + p_2x_2 \leq m\right\}$$

This is the set of all bundles that someone can afford with income m at prices p_1 and p_2.

The **budget line** are all of the bundles that cost exactly m.

$$p_1x_1 + p_2x_2 = m$$

We can transform this by isolating x_2.

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1$$

We get the x_2 intercept immediately. $\frac{m}{p_2}$. The slope is $-\frac{p_1}{p_2}$. The other intercept can be found by plugging 0 in for x_2. The x_1 intercept is $\frac{m}{p_1}$.

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Budget set is described by:

$$(x_1p_1) + (x_2p_2) \leq m$$

"Spends no more than income"

The important part of this budget is the **Budget Line.**

$$x_1p_1 + x_2p_2 = m$$

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1$$

x_2 intercept is $\frac{m}{p_2}$. The slope is $-\frac{p_1}{p_2}$.

We can also get the x_1 intercept by plugging 0 in for x_2.

$$0 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1$$

$$x_1 = \frac{m}{p_1}$$

x_1 intercept is $\frac{m}{p_1}$,x_2 intercept is $\frac{m}{p_2}$. The slope is $-\frac{p_1}{p_2}$.

Interpreting the intercepts:

The x_1 intercept is "how much x_1 can I have if I only buy x_1"

The x_2 intercept is "how much x_2 can I have if I only buy x_2"

These expressions should make sense. $\frac{m}{p_1}$ says "if i spend m on x_1 how many units do I get"

Let's look at another bundle with a similar expression:

$$\left(\frac{\frac{1}{2}m}{p_1}, \frac{\frac{1}{2}m}{p_2}\right)$$

The slope measures how much good 2 I give up to get one more unit of good 1.

This represents the tradeoff I have to make between the goods given their prices.

$$-\frac{p_1}{p_2}$$

What happens to the budget when one of the parameters of the model p_1, p_2, m changes.

Taxes

Quantity tax:

An amount of money you owe to the government **per unit** of some good you buy. Quantity tax of t on good 1.

$$x_1 t + x_1 p_1 + x_2 p_2 = m$$

$$(p_1 + t)\, x_1 + x_2 p_2 = m$$

Ad valorem tax:

A tax on the value of a good purchased:

You owe the government τ times the value of the x_1 you purchase. In the case of Nashville, $\tau \approx 0.09$

$$\tau\,(p_1 x_1) + p_1 x_1 + p_2 x_2 = m$$

For example, if $p_1 = 10$ and $x_1 = 10$ I've spent \$100. If $\tau = 0.09$ then the tax is 9% and I owe \$9 in tax. The total cost of x_1 becomes \$109.

$$((1+\tau)\, p_1)\, x_1 + p_2 x_2 = m$$

The x_1 intercept when the price of x_1 is p_1 if the amount purchased is less than $\bar{x_1}$ and $p_1 + t$ for any units purchased above $\bar{x_1}$.

Let's calculate the cost of buying $\bar{x_1}$ at price p_1:

$$\bar{x_1} p_1$$

The money I have left over is:

$$m - \bar{x_1} p_1$$

The extra x_1 I can buy with this leftover money at the new price of $p_1 + t$ is:

$$\frac{m - \bar{x_1} p_1}{p_1 + t}$$

The total amount I can afford it $\bar{x_1}$ plus this amount:

$$\bar{x_1} + \frac{m - \bar{x_1} p_1}{p_1 + t}$$

Preferences:

Now we will try to model "what a consumer wants"

To represent preferences, we use a "relation". A relation is a set of statements about **pairs** of bundles. You are familiar with some relations already like $\geq$ (greater than or equal to).

$$3 \geq 2$$

$$4 \geq 1$$

A another relation on the set of people might be "Is a sibling of". Let's represent this by s. The following statements are true:

$$Greg\, s\, Christina$$

$$Finn\, s\, Remy$$

In economics, we represent preferences as a relation called the "Preference Relation" $\succsim$.

Suppose a consumer doesn't care about flavor, but just likes more ice cream. We represent bundles such as $(1,1)$ is one scoop of vanilla and one scoop of chocolate. The following formal preference statements are true about this consumer:

$$(1,1) \succsim (0,0)$$

$$(2,1) \succsim (0,2)$$

$$(0,2) \succsim (2,0)$$

The term "preferred" is synonymous to "weakly preferred" and will be true if a consumer either strictly prefers the first bundle to the second (as in the first two cases above) or if they are indifferent such as in the third above.

Contrast this to the term "strictly preferred". The strictly preferred relation is represented by $\succ$.

$$(1,1) \succ (0,0)$$

$$(2,1) \succ (0,2)$$

but the following statement is **not true** about the consumer:

$$(0,2) \succ (2,0)$$

However, the consumer is indifferent between these bundles. The indifference relation is represented by $\sim$. THe following is true.

$$(0,2) \sim (2,0)$$

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Preferences

$$\succsim$$

This is the Weak Preference Relation.

Consumer just wants more ice cream: (Scoops of chocolate, scoops of vanilla).

$$(2,0) \succsim (1,0)$$

From the weak preference relation we can infer strict preference and indifference as well.

We will say $x \succ y$ if and only if $x \succsim y$ and not $y \succsim x$

We will say $x \sim y$ if and only if $x \succsim y$ and $y \succsim x$

For instance:

$$(2,0) \succ (1,0)$$

Since $(2,0) \succsim (1,0)$ but not $(1,0) \succsim (2,0)$.

$$(1,0) \sim (0,1)$$

Since $(1,0) \succsim (0,1)$ and $(0,1) \succsim (1,0)$

Example:

Suppose a consumer has preference over the bundles x, y, z:

$x \succsim y$, $y \succsim z$, $z \succsim y$, $x \succsim z$, $x \succsim x$, $y \succsim y$, $z \succsim z$

What is true about their strict preferences?

$x \succ y, x \succ z$

What is true about their indifference relation?

$y \sim z, z \sim y$

The indifference relation is *symmetric*: whenever $x \sim y$ we also have $y \sim x$.

On the other hand, the strict preference relation is *asymmetric*: if we have $x \succ y$ we don't have $y \succ x$.

The fundamental thing we will work with is the weak preference relation.

Three key assumptions we make about $\succsim$.

1. Reflexive: "every bundle is at least as good as itself"

$$\forall x \in X : x \succsim x$$

For all bundles in the feasible set.

2. Completeness: "Every pair of bundles is comparable"

$$\forall x, y \in X \,\&\, x \neq y : \, (x \succsim y) \vee (y \succsim x) \ \textit{or both}$$

Either one bundle is strictly better than the other or they are indifferent, but they can't say "I don't know". This limits the scope of our models. We should only include bundles that are relevant to a consumers. They should be able to form preferences.

3. Transitivity: "For every three bundles x, y, z, if x is at least as good as y and y is at least as good as z, it must be that x is at least as good as z"

$$\forall x, y, z \in X : x \succsim y \& y \succsim z \implies x \succsim z$$

It is possible to have intransitive preferences. Here is an example:

Choosing mates on a dating app:

Three potential mates:

x. Rich, Very Intelligent, Average Looking

y. Financially Constrained, Genius, Good Looking

z. Moderately Wealthy, Average Intelligence, Best Looking

$y \succ x$ since y is smarter and better looking

$z \succ y$ since z is wealthier and better looking

$x \succ z$ since x is wealthier and smarter than z

These are the only things true about $\succsim$ in this case:

$$y \succsim x, z \succsim y, x \succsim z$$

We have $z \succsim y$ and $y \succsim x$. If this was transitive this should imply $z \succsim x$. This isn't true because x **is strictly better than** z. **These preferences are intransitive.**

While this assumption can fail, it will be reasonable for most of what we do, and it is much easier to "do" economics if preferences are transitive since it will ensure the consumer can make a choice from any budget set.

We construct something called a "choice function". The choice function takes a budget set, and returns a bundle or bundles from the budget set such that that bundle or those bundles are better than everything else in the budget set. The bundle chosen meets this condition: x is "chosen" from budget set B if:

$$\forall x' \in B, x \succsim x'$$

That is, the chosen bundle is at least as good as everything else. If this is not true of the chosen bundle, there is something strictly better. From the set of three mates in the previous example, is there one mate that is at least as good as every other one? No! There is not mate meeting this condition. The choice set is empty.

Transitivity (and completeness) ensures there will always be a "best" or set of "best" bundles. It lets us put things in order. Let's look at our example from the beginning of lecture.

$x \succsim y, y \succsim z, z \succsim y, x \succsim z$,$x \succsim x, y \succsim y, z \succsim z$

This is: Reflexive, Complete, Transitive. Let's rank order the objects. We will put one above the other if is strictly preferred, and put them at the same ranking if they are indifferent. From this weak preference relation we have already extract the strict and weak preferences:

$x \succ y, x \succ z$,$y \sim z, z \sim y$. Here's the ranking:

$$1. x$$

$$2. y, z$$

x is first and y and z are second.

Now let's look at the intransitive relation from the dating app example:

With the intransitive relation $x \succ y, y \succ z, z \succ x$

These are cyclic and that means there is no best. We cannot put them in a rank order.

In order to visualize preference, we use "indifference set" "**indifference curves**". An indifference curve is a set of bundles that are all indifferent to eachother.

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Two distinct indifference curves cannot intersect each other if preferences are complete and transitive.

Since they are distinct, we can find x and some x' such that

$$x \succ x'$$

Because they are on distinct indifference curves.

However, since they intersect there is a bundle $\tilde{x}$ on both curves. Thus, $\tilde{x}$ is indifference to both x and x'

$$\tilde{x} \sim x$$

$$\tilde{x} \sim x'$$

By indifference $x' \succsim \tilde{x}$ and $\tilde{x} \succsim x$.

By transitivity we must have:

$$x' \succsim x$$

However by our original assumption:

$$x \succ x'$$

However, these are contraditory since $x \succ x'$ implies that **not** $x' \succsim x$**.**

We have two statements that contradictory:

$x' \succsim x$ and not $x' \succsim x$.

Perfect Substitutes:

Linear indifference curves. Willingness to trade-off between the goods at **any point** is exactly the same.

Perfect Complements:

Always consume the goods in some fixed proportion. *Left/Right Shoes or Baking Pies.* The indifference curves are L-shaped and the kink-points follow a "ray" through the origin of some slope that represents the ratio the goods are consumed in.

Cobb Douglass:

Convex shaped indifference curves. The more of one good you have, you are willing to give up relatively more to get some of the other good.

Bads:

This occurs when you want less of one of the two (or both) of the goods.

With one bad and one good, the indifference curves slope upwards.

With two bads, the indifference curves again slope downward, but prefrence increases as we move towards the origin.

Well-Behaved Preferences

Monotonicity- "everything is a good"

Weakly Monotonic (Monotonic):

$$(x_1, x_2), \left(x_1^{'}, x_2^{'}\right)$$

If $x_1 \geq x_1'$ and $x_2 \geq x_2^{'}$ then $(x_1, x_2) \succsim \left(x_1^{'}, x_2^{'}\right)$

Furthermore if $x_1 > x_1^{'}$ **and** $x_2 > x_2^{'}$ **then** $(x_1, x_2) \succ \left(x_1^{'}, x_2^{'}\right)$

For instance $(2,3) \sim (3,3)$ and $(4,4) \succ (3,3)$.

Strictly Monotonic

If $x_1 \geq x_1'$ and $x_2 \geq x_2^{'}$ then $(x_1, x_2) \succsim \left(x_1^{'}, x_2^{'}\right)$

Furthermore if $x_1 > x_1^{'}$ **or** $x_2 > x_2^{'}$ **then** $(x_1, x_2) \succ \left(x_1^{'}, x_2^{'}\right)$

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Monotonicity:

“More is better”.

Weakly monotonic (monotonic):

If I have at least as much of everything, I am at least as well off and if I have strictly more of everything I am strictly better off.

Strictly monotonic:

If I have at least as much of everything, I am at least as well off and if I have strictly more of *anything* I am strictly better off.

Under both assumptions we know:

$$(2, 2) \succ (1, 1)$$

Under both assumptions we know:

$$(2, 1) \succsim (1, 1)$$

We only know this for sure under **strictly monotonic preferences**:

$$(2, 1) \succ (1, 1)$$

If preferences are only weakly monotonic it is possible:

$$(2, 1) \sim (1, 1)$$

Perfect complements are an example of a weakly but not strictly monotonic preference.

Perfect substitutes are an example of strictly monotonic preferences.

The indifference curves of monotonic preferences are never upward sloping, but if they are only weakly monotonic they may have points of zero or undefined slope.

Convexity:

Intermediate bundles are better than extreme bundles.

Pick two points that are indifferent to eachother.

$$(x_1, x_2) \sim \left(x_1^{'}, x_2^{'}\right)$$

Take a convex combination (mixture) of the two bundles. Pick a t between 0 and 1.

$$\left(tx_1 + (1-t)\,x_1^{'}, tx_2 + (1-t)\,x_2^{'}\right)$$

If we pick $t = 0$ we get

$$\left(0x_1 + (1)\,x_1^{'}, 0x_2 + (1)\,x_2^{'}\right) = \left(x_1^{'}, x_2^{'}\right)$$

Pick $t = 1$ we get:

$$(x_1, x_2)$$

If we pick $t = \frac{1}{2}$

$$\left(\frac{1}{2}x_1 + \frac{1}{2}x_1^{'}, \frac{1}{2}x_2 + \frac{1}{2}x_2^{'}\right)$$

This is half-way between the two on a straight line between them.

For t from 0 to 1 we all the bundles on a stright line between them.

Weakly Convex preferences (Convex Prefrences). For any two indifference bundles $(x_1, x_2) \sim \left(x_1^{'}, x_2^{'}\right)$ and for all $t \in [0, 1]$

$$\left(tx_1 + (1-t)\,x_1^{'}, tx_2 + (1-t)\,x_2^{'}\right) \succsim (x_1, x_2)\, or \left(x_1^{'}, x_2^{'}\right)$$

Strictly Convex preferences (Convex Prefrences). For any two indifference bundles $(x_1, x_2) \sim \left(x_1^{'}, x_2^{'}\right)$ and for all $t \in (0, 1)$

$$\left(tx_1 + (1-t)\,x_1^{'}, tx_2 + (1-t)\,x_2^{'}\right) \succ (x_1, x_2)\, or \left(x_1^{'}, x_2^{'}\right)$$

Strictly convex preferences have indifference curves that bend outward strictly. Weakly convex preference may have flat spots.

Cobb Douglass is strictly convex. Perfect substitutes are weakly convex. Perfect complements are only weakly convex because if we choose two bundles on the same "leg" of the indifference curve, a line between them lies on the indifference curve.

If preferences are complete, transitive, and monotonic, an optimal bundle must occur at a point on the budget line where the indifference through that point "just" touches the budget line.

Now we know where to look for optimal bundles.

How do we represent preferences in a way that will allow us to find the slopes of the indifference curves to find optimal bundles?

Utility Function.

A mathematical representation of preferences.

$u(2,2) = 20$

$u(1,1) = 10$

$u(0,1) = 5$

$u(1,0) = 5$

$u(0,0) = 0$

$u(2,2) = 4$

$u(1,1) = 2$

$u(0,1) = 1$

$u(1,0) = 1$

$u(0,0) = 0$

A utility function is a function that assigns every bundle a number with the property that better bundles get higher numbers.

The function u which maps bundles into real number is a **Utility function** if:

$u(x_1, x_2) \geq u\left(x_1^{'}, x_2^{'}\right)$ if and only if $(x_1, x_2) \succsim \left(x_1^{'}, x_2^{'}\right)$

Many different utility functions can represent the same preferences.

Let's write some utility functions that represent our families of preferences.

Perfect substitutes:

$$u(x_1, x_2) = 2x_1 + x_2$$

We can find equations for particular indifference curves by plugging some number in for u

$$10 = 2x_1 + x_2$$

$$x_2 = 10 - 2x_1$$

We can find the slope of an indifference curve at any point by taking the ratio of the partial derivatives of the utility function.

$$MRS = -\frac{\frac{\partial(u(x_1,x_2))}{\partial x_1}}{\frac{\partial(u(x_1,x_2))}{\partial x_2}}$$

$$MRS = -\frac{\frac{\partial(2x_1+x_2)}{\partial x_1}}{\frac{\partial(2x_1+x_2)}{\partial x_2}} = -2$$

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$$MRS = -\frac{\frac{\partial(u(x_1,x_2))}{\partial x_1}}{\frac{\partial(u(x_1,x_2))}{\partial x_2}}$$

Slope of an indifference curve at any point.

This is interpreted as the rate a consumer is willing to trade off x_2 to get more x_1.

Example:

$u(x_1, x_2) = x_1 + x_2$. Perfect substitutes.

$$MRS = -\frac{\frac{\partial(x_1+x_2)}{\partial x_1}}{\frac{\partial(x_1+x_2)}{\partial x_2}} = -1$$

Common Utility Representations.

Perfect Substitutes- $u(x_1, x_2) = ax_1 + bx_2$

$$MRS = -\frac{\frac{\partial(ax_1+bx_2)}{\partial x_1}}{\frac{\partial(ax_1+bx_2)}{\partial x_2}} \quad = \quad -\frac{a}{b}$$

Cobb Douglass- $u(x_1, x_2) = x_1^\alpha x_2^\beta$

$$MRS = -\frac{\frac{\partial\left(x_1^\alpha x_2^\beta\right)}{\partial x_1}}{\frac{\partial\left(x_1^\alpha x_2^\beta\right)}{\partial x_2}} = -\frac{\alpha x_1^{\alpha-1}x_2^\beta}{\beta x_1^\alpha x_2^{\beta-1}} = -\frac{\alpha x_1^\alpha \frac{1}{x_1} x_2^\beta}{\beta x_1^\alpha x_2^\beta \frac{1}{x_2}} = -\frac{\alpha \frac{1}{x_1}}{\beta \frac{1}{x_2}} = -\frac{\alpha}{\beta}\frac{x_2}{x_1}$$

$$\alpha = \beta = 1, MRS = -\frac{x_2}{x_1}$$

$(10, 10), MRS = -\frac{10}{10} = -1$.

$(100, 10), MRS = -\frac{10}{100} = -0.1$ *not willing to give up much x_2 to get more x_1.*

$(10, 100), MRS = -\frac{100}{10} = -10$ *willing to give up a lot x_2 to get more x_1.*

Quasi-Linear- $u(x_1, x_2) = f(x_1) + x_2$

Common Example:

$ln(x_1) + x_2$

$$MRS = -\frac{\frac{\partial(ln(x_1)+x_2)}{\partial x_1}}{\frac{\partial(ln(x_1)+x_2)}{\partial x_2}} = -\frac{\frac{1}{x_1}}{1} = -\frac{1}{x_1}$$

How much x_2 I will give up to get more x_1 is decreasing in the amount of x_1 I have, but **does not depend on how much x_2 I have.**

Another Example:

$\sqrt{x_1} + x_2$

Perfect Complements- $u(x_1, x_2) = min\{ax_1, bx_2\}$

Left\Right Shoes. $min\{x_1, x_2\}$

$u(3,2) = min\{3,2\} = 2$

$u(2,2) = min\{2,2\} = 2$

Pies (2 apples 1 crust).

This is the wrong way to represent this: $u(x_1, x_2) = min\{2x_1, x_2\}$

$(2,1), (2,2)$ both giving me 1 pies.

$$min\{4,1\} = 1$$

$$min\{4,2\} = 2$$

This is the right way. $u(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$

With 4 apples, the most pies I could make is 2. # I have divided my number I need per pie. $\frac{4}{2} = 2$.

With 2 crusts, the most pies I could make is 2.

Multiple Utility Functions can Represent the Same Preferences.

Monotonic Transformations. *If we take a strictly increasing function of any utility function, the resulting function represents the same preferences.*

Suppose we have preferences over ice cream where the flavor does not matter.

$u(x_1, x_2) = x_1 + x_2$

$\tilde{u}(x_1, x_2) = x_1 + x_2 + 10$

$\hat{u}(x_1, x_2) = 10(x_1 + x_2)$

$$(1,2) \succ (1,1)$$

$$u(1,2) = 3, u(1,1) = 2$$

$$\tilde{u}(1,2) = 13, u(1,1) = 12$$

$$\hat{u}(1,2) = 30, u(1,1) = 20$$

All of these utility functions, represent the same tradeoff. **They are monotonic transformations of each other.**

$$u'(x_1, x_2) = \log\left(14\sqrt{10(x_1 + x_2) - 1} + 8\right)$$

This represents the same preferences as $u(x_1, x_2) = x_1 + x_2$

$$MRS = -\frac{\frac{\partial\left(\log\left(14\sqrt{10(x_1+x_2)-1}+8\right)\right)}{\partial x_1}}{\frac{\partial\left(\log\left(14\sqrt{10(x_1+x_2)-1}+8\right)\right)}{\partial x_2}} \quad = \quad -1$$

Same MRS, Same Preferences.

7 Class 7- 2/9/2022

If a consumer if doing something optimal, they must choose a bundle that is on an indifference curve that never passes into the budget set.

Tangency. The slope of the indifference curve is equal to the slope of the budget equation.

If the slope of the indifference curve was not equal to the slope of the budget equation then either I'm willing to give up more x_2 than I have to to get more x_1 or the opposite. Either way, the choice isn't optimal.

Unless I can't give up any more x_2 (or x_1). This condition **has to be true at any optimum.**

We have two conditions to check to find optimal bundles.

1) Find all the points where $MRS = -\frac{p_1}{p_2}$

2) Find all the points on the budget equation. (spent all money)

Any point meeting both conditions is a candidate for being optimal. Often, there's only one point meeting these conditions.

Example:

$$u(x_1, x_2) = x_1 x_2$$

Prices are $p_1 = 1$ and $p_2 = 2$, and $m = 100$.

1) If a bundle is optimal and involves consuming some of both goods, it must be that:

$$MRS = -\frac{p_1}{p_2}$$

Tangency Condition:

$$-\frac{x_2}{x_1} = -\frac{1}{2}$$

$$2x_2 = x_1$$

Budget Condition:

$$1x_1 + 2x_2 = 100$$

Plug the tangency condition into the budget condition:

$$x_1 + x_1 = 100$$

Consumer demand is:

$$\boxed{x^* = (50, 25)}$$

Example:

$$u(x_1, x_2) = x_1 x_2$$

Prices are p_1 and p_2, and m.

$$MRS = -\frac{p_1}{p_2}$$

Tangency Condition:

$$-\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

$$x_2 p_2 = x_1 p_1$$

$x_1 p_1$ is the amount of money spent on good 1. $x_2 p_2$ is the amount of money spent on good 2.

If the consumer is doing something optimal, they spend the same amount of money on both goods.

$$p_1 x_1 + p_2 x_2 = m$$

Plugging in the tangency condition:

$$p_2 x_2 + p_2 x_2 = m$$

$$2(p_2 x_2) = m$$

$$x_2 = \frac{\frac{1}{2}m}{p_2}$$

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

Marshallian Demand:

$$x^*\left(p_1, p_2, m\right) = \left(\frac{\frac{1}{2}m}{p_1}, \frac{\frac{1}{2}m}{p_2}\right)$$

In this particular case, they spend half their income $\frac{1}{2}m$ on both goods.
What if we had a difference cobb douglass utility function?

$$u\left(x_1, x_2\right) = x_1^{\alpha} x_2^{\beta}$$

$$x^*\left(p_1, p_2, m\right) = \left(\frac{\frac{\alpha}{\alpha+\beta}m}{p_1}, \frac{\frac{\beta}{\alpha+\beta}m}{p_2}\right)$$

If we had:

$$u\left(x_1, x_2\right) = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}}$$

$$x^*\left(p_1, p_2, m\right) = \left(\frac{\frac{2}{3}m}{p_1}, \frac{\frac{1}{3}m}{p_2}\right)$$

Example of perfect substitutes.

$$u\left(x_1, x_2\right) = x_1 + x_2$$

$$p_1 = 1, p_2 = 2, m = 100$$

Our expectation is that this consumer should only buy x_1.

$$\left(\frac{m}{p_1}, 0\right)$$

Let's suppose we did not know this up-front and looked at the tangency condition.

$$MRS = -\frac{\frac{\partial(x_1+x_2)}{\partial x_1}}{\frac{\partial(x_1+x_2)}{\partial x_2}} = -1$$

$$-1 = -\frac{1}{2}$$

The tangency condition **can never be met**. Yet, the tangency condition is required to be true for an *interior* optimum. The only thing left is a boundary solution. That is: consume all x_1 or all x_2.

Check the utility of the two intercepts.

$$u\left(\frac{m}{p_1},0\right)=\frac{100}{1}=100$$

$$u\left(0,\frac{m}{p_2}\right)=\frac{100}{2}=50$$

Since I get more utility from buying only x_1, that is the optimal solution.

Let's try another one:

$$u\left(x_1,x_2\right)=2x_1+3x_2$$

$$p_1=2,p_2=4,m=100$$

$$u\left(50,0\right)=2*\left(50\right)=100$$

$$u\left(0,25\right)=3*\left(25\right)=75$$

Buy only x_1 $\left(50,0\right)$.

$$u\left(x_1,x_2\right)=2x_1+4x_2$$

$$p_1=2,p_2=4,m=100$$

$$-\frac{1}{2}=-\frac{1}{2}$$

Slope of the indifference curve is equal to the slope of the budget equation. The consume is willing to buy any bundle that costs m.

$$u\left(50,0\right)=2\left(50\right)=100$$

$$u\left(0,25\right)=4\left(25\right)=100$$

The utility of the endpoints (buying only x_1 or only x_2) is the same, and so **any bundle that costs m is optimal.**

Perfect Complements:

The only case we will run into where this procedure will fail is for perfect complements. Because the MRS is not defined.

For left and right shoes, there is not reason to buy more left shoes than right shoes, or more right shoes than left shoes. Any optimal bundle must have $x_1=x_2$.

For pies, there's no reason to buy more than 2 times the number of apples than I have crusts or more than one half the number of crusts than I have apples.

In either condition, we've wasted money on goods that don't contribute to my utility.

No Waste Condition.

Always consume at the kink of the indifference curve.

Left right shoes:

$$min\left\{x_1, x_2\right\}$$

$$x_1 = x_2$$

2 apples, 1 crust

$$min\left\{\frac{1}{2}x_1, x_2\right\}$$

$$\frac{1}{2}x_1 = x_2$$

The no-waste condition replaces the tangency condition in solving for the optimal bundle.

Example:

$$u\left(x_1, x_2\right) = min\left\{\frac{1}{2}x_1, x_2\right\}$$

$p_1 = 1, p_2 = 2, m = 30.$

No waste condition:

$$\frac{1}{2}x_1 = x_2$$

$$x_1 = 2x_2$$

Budget condition:

$$x_1 + 2x_2 = 30$$

Plug the first into the second:

$$2x_2 + 2x_2 = 30$$

$$4x_2 = 30$$

$$x_2 = \frac{30}{4}$$

$$x_1 = \frac{30}{2}$$

$$\left(\frac{30}{2}, \frac{30}{4}\right)$$

Try this one:
Find the **marshallian demand:**

$$u\left(x_1, x_2\right) = min\left\{\frac{1}{2}x_1, x_2\right\}$$

$p_1, p_2, m.$
Example Quasi-Linear

$$u\left(x_1, x_2\right) = \log\left(x_1\right) + x_2$$

p_1, p_2, m
Tangency Condition:

$$-\frac{1}{x_1} = -\frac{p_1}{p_2}$$

$$x_1 p_1 = p_2$$

Budget Equation:

$$x_1 p_1 + x_2 p_2 = m$$

Plug one into the other:

$$p_2 + x_2 p_2 = m$$

$$p_2\left(1 + x_2\right) = m$$

$$x_2 = \frac{m}{p_2} - 1$$

$$x_1 = \frac{p_2}{p_1}$$

Marshallian Demand:

$$x^*(p_1, p_2, m) = \left(\frac{p_2}{p_1}, \frac{m}{p_2} - 1\right)$$

Suppose we have $p_1 = 1, p_2 = 1, m = 100$

$$(1, 99)$$

There's a problem here: Suppose we have $p_1 = 1, p_2 = 200, m = 100$

$$\left(200, -\frac{1}{2}\right)$$

This can't be right. This is impossible.
Instead the consumer will choose the boundary point $\left(\frac{m}{p_1}, 0\right)$ when the bundle the would like involves a negative amount of x_2.
There is no bundle that meets the tangency condition and is on the budget equation. Both of these conditions **must be true** at an interior optimum.
There is no interior optimum at these prices and income.
There must be a boundary solution.

$$u(100, 0) = \log(100) \approx 4.60517$$

$$u\left(0, \frac{100}{200}\right) = u\left(0, \frac{1}{2}\right) = \frac{1}{2}$$

Since no interior solution can be optimal, the best boundary solution must be optimal. The best boundary solution is

$$(100, 0)$$

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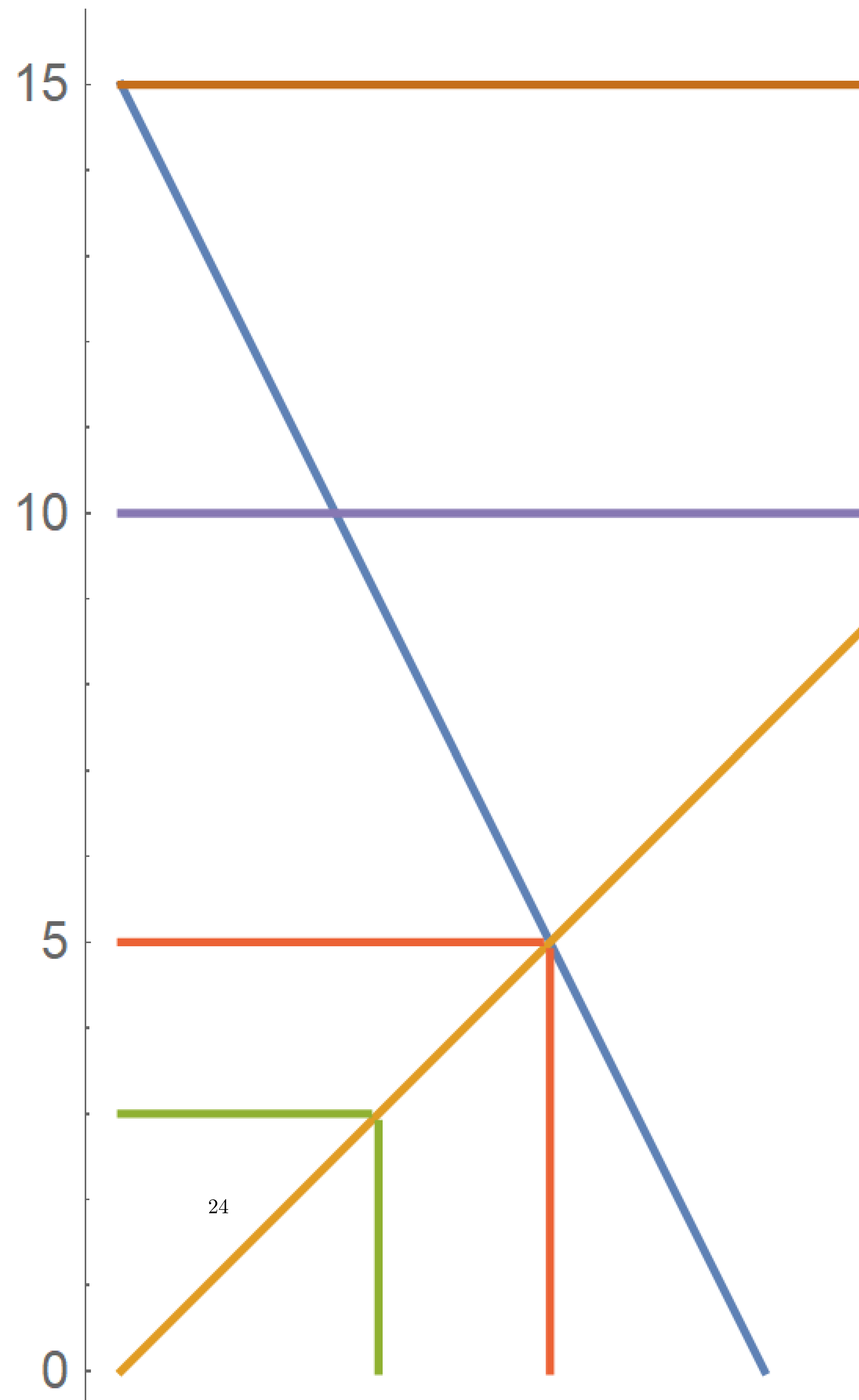
The utility and the budget are:

$$u(x_1, x_2) = min\{x_1, x_2\}$$

Budget: $p_1 = 2$, $p_2 = 1$, $m = 15$

$$2x_1 + 1x_2 = 15$$

Let's plot this problem:


15
10
5
0

No waste condition:

$$x_1 = x_2$$

Budget Equation:

$$2x_1 + 1x_2 = 15$$

Let's solve these together:

$$3x_1 = 15$$

$$x_1 = 5, x_2 = 5$$

Let's do the **max** version of this problem.
The utility and the budget are:

$$u(x_1, x_2) = max\{x_1, x_2\}$$

Budget: $p_1 = 2$, $p_2 = 1$, $m = 15$

$$2x_1 + 1x_2 = 15$$

In this case, plotting indifference shows us that the optimal bundle is $(0, 15)$.

Demand
Marshallian:

$$x_1(p_1, p_2, m), x_2(p_1, p_2, m)$$

For instance for the utility function: $u = x_1x_2$ we've seen the marshallian demands are:

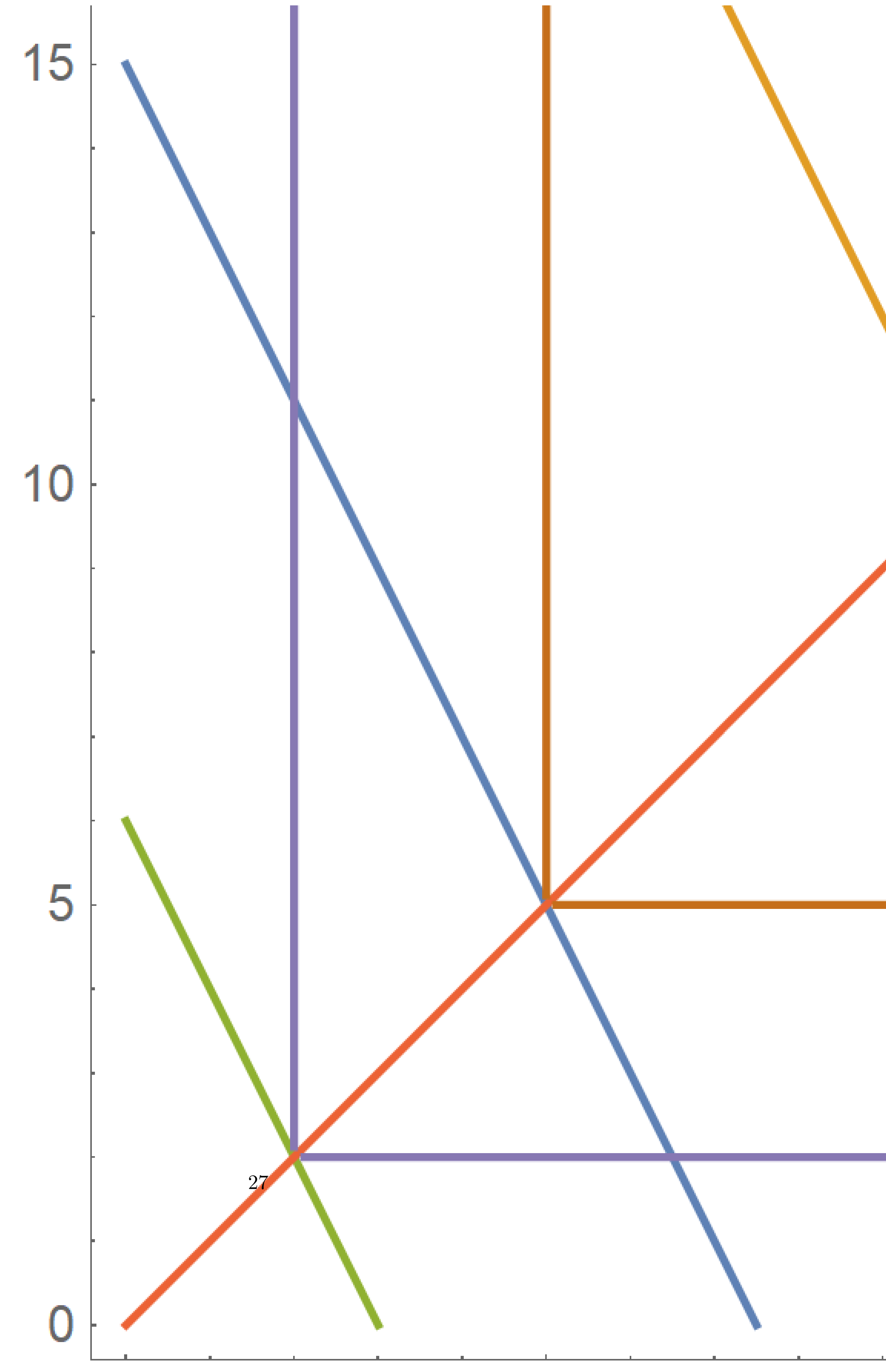
$$\left(\frac{\frac{1}{2}m}{p_1}, \frac{\frac{1}{2}m}{p_2}\right)$$

How does demand for a good chance when prices or income changes.
Changes in income.

$$\frac{\frac{1}{2}m}{p_1}$$

Since income is only in the numerator of this function, as income increases, demand increases as well for any fixed prices. We can also do this formally, by taking a derivative.

$$\frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial m} = \frac{1}{2p_1} > 0$$


15
10
5
0

Inferior good. *When income increases, demand decreases.*

A can't always be inferior. The reason is, to decrease, it has to have increased at some point. Furthermore, when $m = 0$ demand has to be 0.

For instance it might be normal for low levels of income and inferior for high levels of income.

Two graphs associated with the behavior of goods with respect to income.

Income Offer Curve: A plot of demand **bundles** of both goods for fixed prices as income changes.

Example:

$$u = min\{x_1, x_2\}$$

Budget:

$$2x_1 + 1x_2 = m$$

Example of Cobb Douglass Demand:

$$u = x_1 x_2$$

$$2x_1 + 1x_2 = m$$

$$\left(\frac{\frac{1}{2}m}{2}, \frac{\frac{1}{2}m}{1}\right)$$

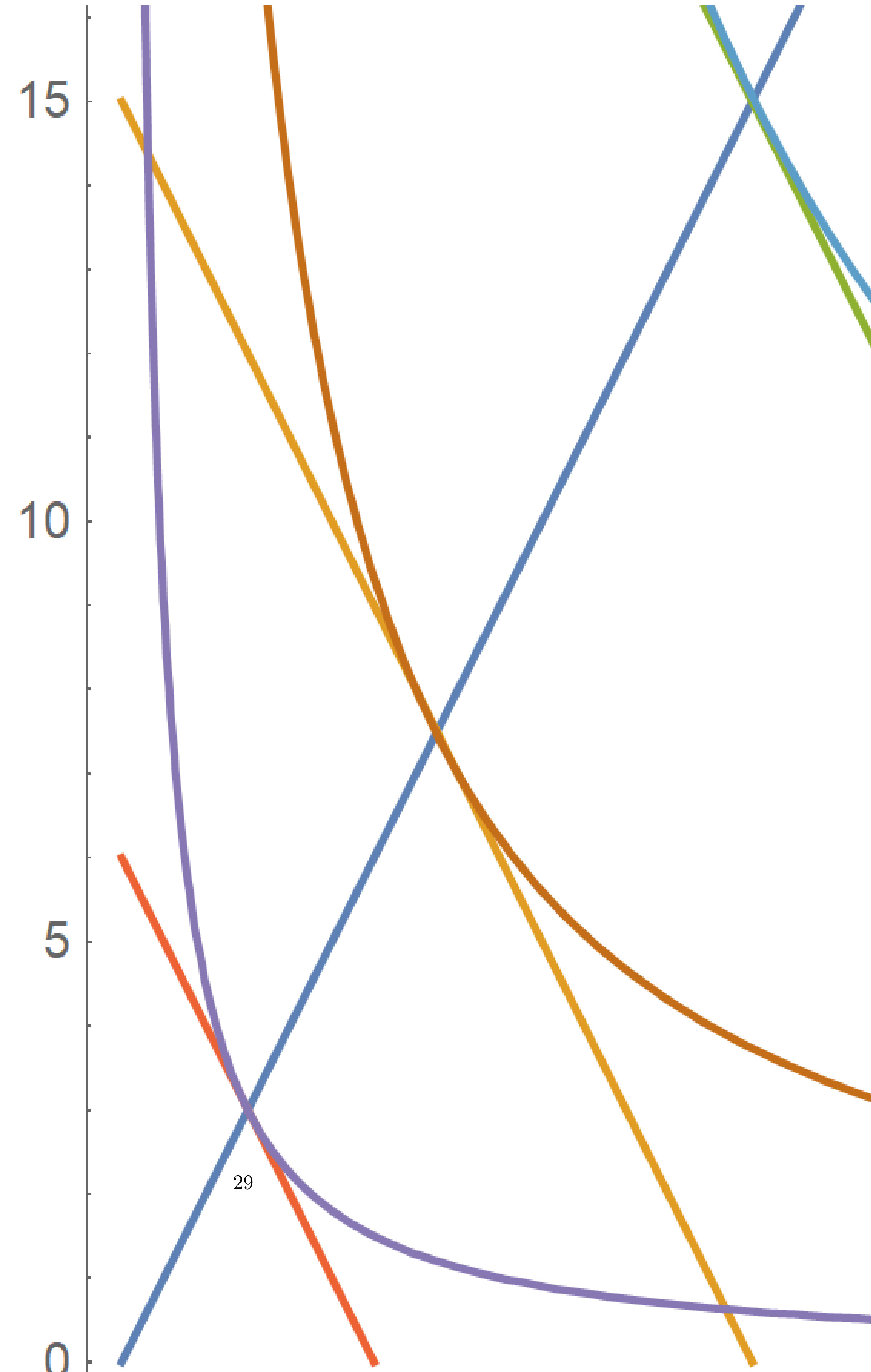
$$x_1 = \frac{\frac{1}{2}m}{2}$$

$$x_2 = \frac{\frac{1}{2}m}{1}$$

$$4x_1 = m$$

$$x_2 = 2x_1$$

In this case the income offer curve is a line with slope 2 coming from the origin.


15
10
5
0

Engle Curve: A plot of the demand for one good for fixed prices as income changes.

Put income on the vertical axis and demand on the horizontal axis. This really answers the question: "for a particular level of demand for some good, what income would be responsible for that amount of demand?"

For example:

$$x_1\left(2,1,m\right)=\frac{\frac{1}{2}m}{2}$$

$$m=4x_1$$

This is a line with slope of positive 4.

$$m=2x_2$$

Line with slope of positive 2.

When engle curves are linear, every extra dollar of income will get spent the same way.

$$\left(\frac{\frac{1}{2}m}{2},\frac{\frac{1}{2}m}{1}\right)$$

Give them an extra dollar, they will spend 50 cents on good 1 and 50 cents on good 2. This buys them an extra $\frac{1}{4}$ unit of good 1 and $\frac{1}{2}$ unit of good 2.

Changes in Own Price

Ordinary- When the price of a good goes up, the demand for that good goes down.

Giffen- When the price of a good goes up, the demand for that good goes up.

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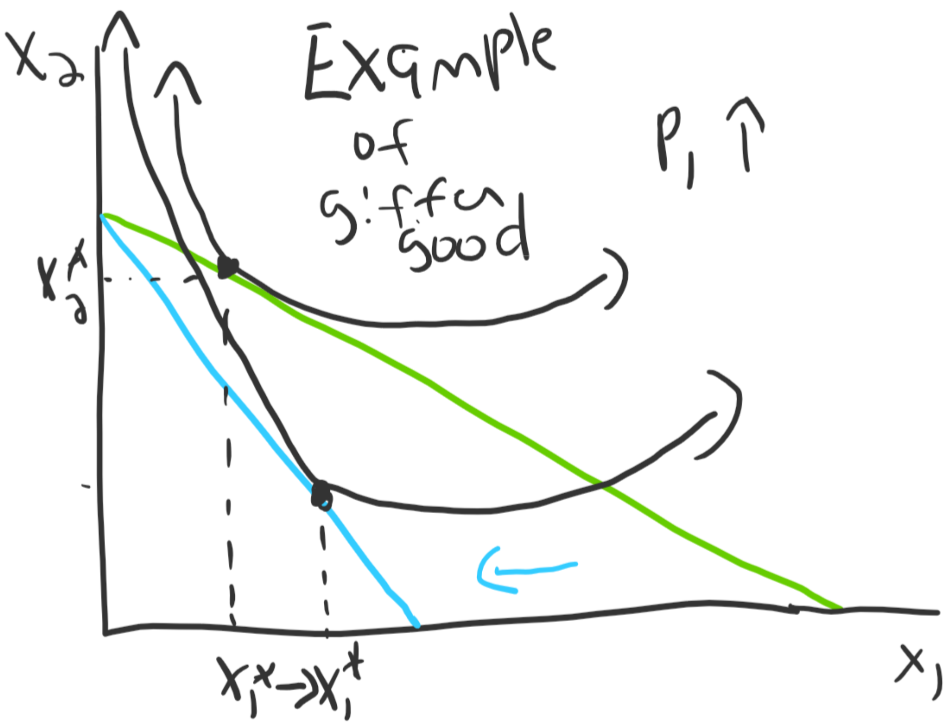


Figure 1: Indifference curves and budget for a "giffen" good. Note that as the price p_1 increases from the green to blue budget, the optimal amount of x_1 increases.

Price Offer Curve. A plot of the optimal bundles as one price changes and the other price and income stay fixed.

Suppose we have utility x_1x_2. The demand is:

$$\left(\frac{\frac{1}{2}m}{p_1}, \frac{\frac{1}{2}m}{p_2}\right)$$

So let's fix $m = 20$, $p_2 = 2$

$$\left(\frac{10}{p_1}, 5\right)$$

Suppose we have perfect complements preferences. Utility function $min\left\{x_1, x_2\right\}$.

$$\left(\frac{m}{p_1+p_2}, \frac{m}{p_1+p_2}\right)$$

So let's fix $m = 20$, $p_2 = 2$

$$\left(\frac{20}{p_1+2}, \frac{20}{p_1+2}\right)$$

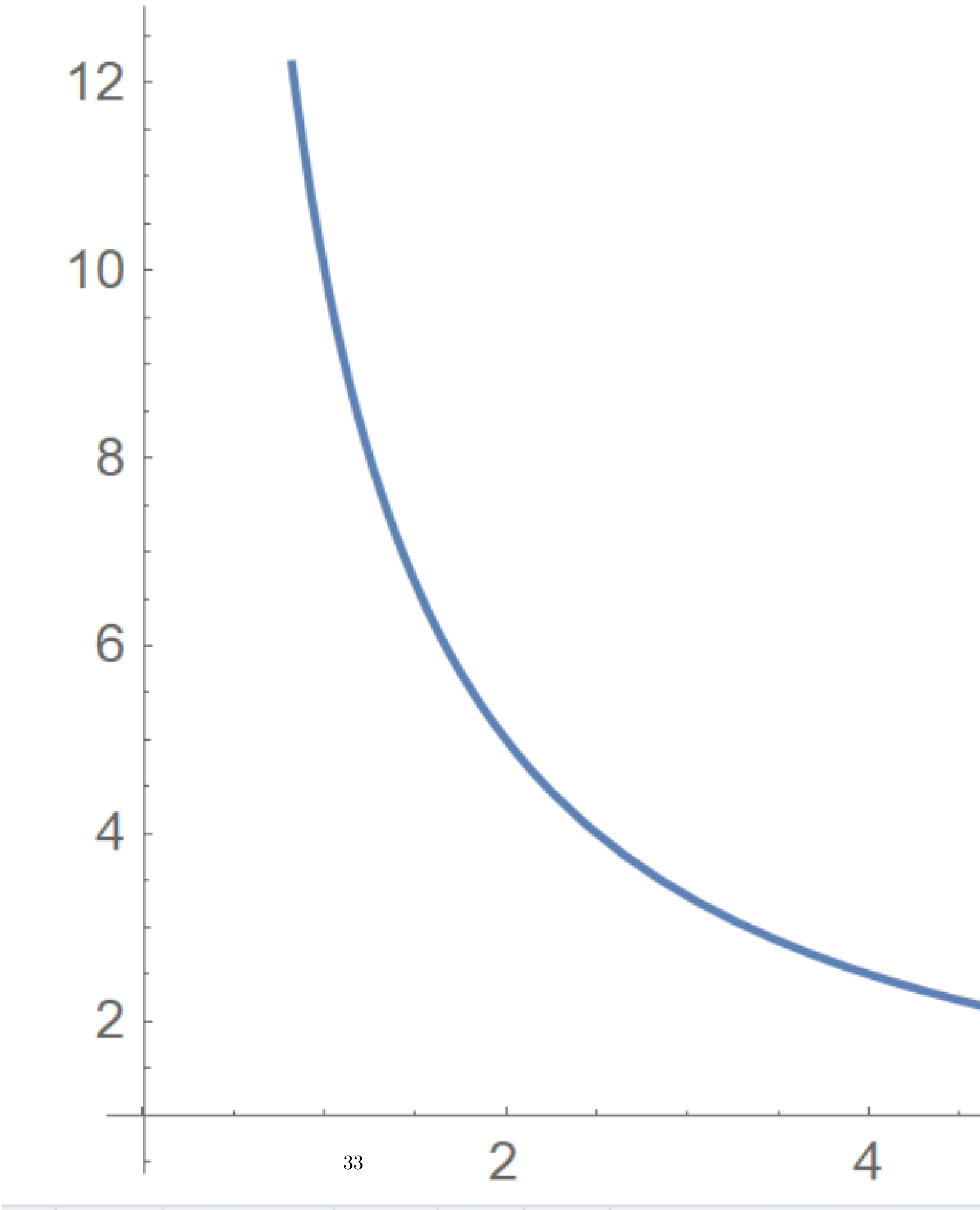
Demand Curve. A plot of the optimal amount of a good as that good's price changes. Plot p on the vertical axis and demand on the horizontal axis.

In the previous cobb douglass example, demand for good 1 is:

$$x_1 = \frac{10}{p_1}$$

To plot this, we need to isolate p_1. This actually called the "inverse demand". At what price is x_1 amount of good 1 demanded.

$$p_1 = \frac{10}{x_1}$$

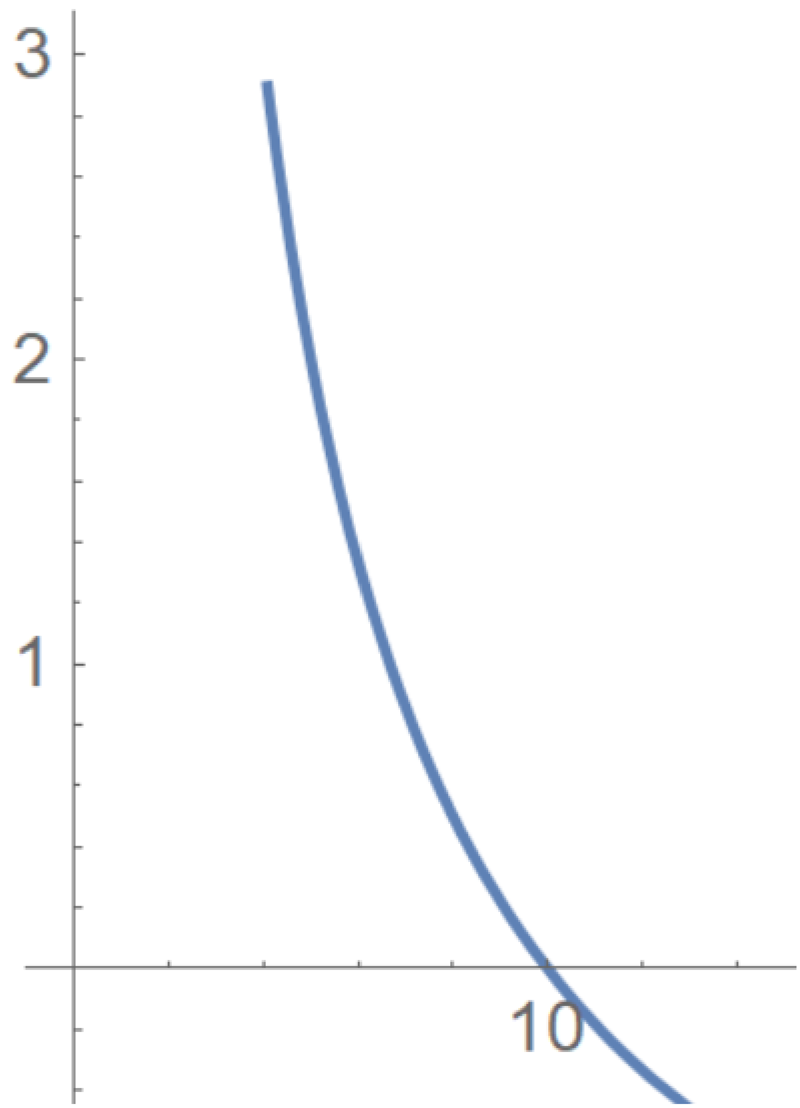

12
10
8
6
4
2
2
4

For the example of perfect complements. Demand for good 1 is:

$$x_1 = \frac{20}{p_1 + 2}$$

The inverse demand is:

$$p_1 = \frac{20}{x_1} - 2$$



Changes in the demand for one good when we change the price of the **other good**.

Two possibilities:

Substitutes: When the price of the other good goes up, demand goes up.

Complements: When the price of the other good goes up, demand goes down.

If when the price of the the other good goes up, demand does not change, we say they are neither.

For example: For cobb douglass $u = x_1 x_2$

$$\left(\frac{\frac{1}{2}m}{p_1}, \frac{\frac{1}{2}m}{p_2} \right)$$

Suppose we have perfect complements preferences. Utility function $min\left\{x_1, x_2\right\}$.

$$\left(\frac{m}{p_1 + p_2}, \frac{m}{p_1 + p_2} \right)$$

These are complements (as expected) because as the price of the other good goes up, demand goes down.

Let's take a derivative of x_1 with respect to p_2 to find this formally.

$$\frac{\partial \left(\frac{m}{p_1 + p_2} \right)}{\partial \left(p_2 \right)} = -\frac{m}{\left(p_1 + p_2 \right)^2}$$

m, p_1, p_2 are always positive.

$$-\frac{m}{\left(p_1 + p_2 \right)^2} < 0$$

Formally, this shows that the goods are complements because as p_2 goes up, x_1 goes down.

10 Class 10- 2/21/2022

Two effects:

Substitution effect. When the price of a good goes up, I buy other stuff and so demand for the good goes down. **The substitution effect is always negative. When a price goes up, demand goes down. Law of demand.**

Income effect. When prices goes up, your effective income goes down and this will change demand based on whether goods are normal or inferior.

We want to decompose a "total effect" into these two effects.

The way we decompose this is through a "thought experiment".

The price of a good changes. We will imagine giving the consumer enough extra money so that they can afford the old bundle (what they bought before the price change) and then ask what they demand at the new prices but this **compensated income.** Whatever the consumer demands under this thought experiment budget can't be due to income effect.

Example: suppose utility is $u = x_1 x_2$. $p_1 = 1$, $p_2 = 1$ and income $m = 10$.

$$x_1 = \frac{\frac{1}{2}m}{p_1}, x_2 = \frac{\frac{1}{2}m}{p_2}$$

$$x_1 = 5, x_2 = 5$$

Let's suppose the price of x_1 goes up to $p_1 = 2$.

$$x_1 = 2.5, x_2 = 5$$

The total change in demand for x_1 is 2.5.

To find what part of this change is due to substitution? The first step is to find the **compensated income**. What income will allow the consumer to buy the bundle $(5, 5)$- the original bundle?

$$2\,(5) + 1\,(5) = \$15$$

Needs \$15 to buy the old bundle at the new prices. What do they demand under the new prices with this compensated income? The thought experiment budget equation is:

$$2x_1 + 1x_2 = 15$$

$$x_1 = \frac{\frac{1}{2}\,(15)}{2}, x_2 = \frac{\frac{1}{2}\,(15)}{1}$$

$$x_1 = 3.75, x_2 = 7.5$$

Notice that x_1 is lower than it was originally; by $5 - 3.75 = 1.25$ units.

This can't be due to the income effect, it is the substitution effect.

$$SE = 1.25$$

Demand decreases by 1.25 due to substitution. To find the income effect use the equatoin:

$$TE = SE + IE$$

The total effect is 2.5, the substitution effect is 1.25 so the income effect must be:

$$IE = 2.5 - 1.25 = 1.25$$

This process is called the Slutsky Decomposition.

Perfect Complements Example:

$p_1 = 1$, $p_2 = 1$ and income $m = 10$.

$$u(x_1, x_2) = min\{x_1, x_2\}$$

No waste condition:

$$x_1 = x_2$$

Budget equation:

$$x_1 + x_2 = 10$$

Original Demand:

$$x_1 = 5, x_2 = 5$$

After the prices change, what is the new demand?

If we had solved for demand generically we would have gotten:

$$x_1 = \frac{m}{p_1 + p_2}, x_2 = \frac{m}{p_1 + p_2}$$

Now $p_1 = 2$

$$x_1 = \frac{10}{3}, x_2 = \frac{10}{3}$$

Total change in demand for $x_1 = 5 - \frac{10}{3} = \frac{5}{3}$. Demand for x_1 decreases by $\frac{5}{3}$ due to this change in price. This is the **total effect.**

Compensated income the cost of the old bundle under the new prices:

$$2(5) + 1(5) = 15$$

What do they choose under this compensated income at the new prices:

$$x_1 = 5, x_2 = 5$$

The substitution effect is the original demand minus this demand which is 0. There is no substitution effect. The total change is due to income.

$$TE = \frac{5}{3}, SE = 0, IE = \frac{5}{3}.$$

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Exogenous income. (assumed rather than modeled). m
Endogenous income. (moving part of the model).
Money -> Endowment of Goods

(ω_1, ω_2)- endowment of good 1 and good 2.
This is the bundle the consumer starts the model with.
x_1 is apples, x_2 is crusts.
Apple farmer starts with the endowment $(10, 0)$.
The farmer has 10 apples and zero crusts.
Endowment is always in the budget set.
The old equation for the budget line:

$$p_1x_1 + p_2x_2 = m$$

The new equation sets the amount spent to the **v value** (cost) of the endowment.

$$p_1x_1 + p_2x_2 = (p_1\omega_1 + p_2\omega_2)$$

Suppose we have apple farmer who has endowment $(10, 0)$ prices are $p_1 = 2$, $p_2 = 1$ and preferences are represented by:

$$u = min\left\{\frac{1}{2}x_1, x_2\right\}$$

Budget equation:

$$2\,(x_1) + 1\,(x_2) = 2\,(10) + 1\,(0)$$

$$2x_1 + x_2 = 20$$

Two conditions:
No waste condition:

$$\frac{1}{2}x_1 = x_2$$

Budget condition:

$$2x_1 + x_2 = 20$$

Plug one condition into the other:

$$2x_1 + \frac{1}{2}x_1 = 20$$

$$\frac{5}{2}x_1 = 20$$

$$x_1 = 8$$

$$x_2 = 4$$

$$(8, 4)$$

Notice that the demand is not identical to the endowment. In this case, the demand for apples is less than the endowment.

Net Seller of apples.

Net Buyer of crusts.

If you are a net buyer of one good, you are a net seller of the other.

This $(8, 4)$ is the consumers **gross demand.**

The difference between the **gross demand** and the **endowment** is the **net demand.**

$$(8, 4) - (10, 0) = (-2, 4)$$

When net demand is negative, the consumer is a net seller, when net demand is positive, they are a net buyer.

Price increases/decreases can make consumer better off or worse off.

If a consumer is a net buyer of a good and price goes down, they remain a net buyer and will be strictly better off.

We know they are better off because there are bundles that are affordable and contain more of both goods.

If a consumer is a net seller of a good and the price goes up, they remain a net seller and will be strictly better off.

x_1, x_2 ω_1, ω_2

x_1 consumption today (how much \$ I'll spend today)

x_2 consumption in the future (... \$ I'll spend in the future)

ω_1 income today

ω_2 income in the future.

If you want to borrow money,

$$x_1 - \omega_1 > 0$$

Borrow today is a buyer of consumption in period 1, they must be a seller of consumption in period 2.

$$x_2 - \omega_2 < 0$$

Budget equation for borrowing looks like this:

Size of the loan I take out today:

$$(x_1 - \omega_1)$$

You have to pay back:

$$(x_1 - \omega_1) + r\,(x_1 - \omega_1) = (1 + r)\,(x_1 - \omega_1)$$

If $r = 0.05$ and I borrow \$1000, I pay back \$1050.

How much can I consume tomorrow?

$$x_2 = \omega_2 - (1 + r)\,(x_1 - \omega_1)$$

$$x_2 = \omega_2 - (1 + r)\,x_1 + (1 + r)\,\omega_1$$

$$(1 + r)\,x_1 + x_2 = (1 + r)\,\omega_1 + \omega_2$$

Future value version of the budget.

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c_1 c_2 (consumption today c_1 and in the future c_2). Income today is m_1 in the future it is m_2.

I can save money. Any money saved will earn me r interest in period 2.

I can borrow money. Any money borrow has to be paid back in period 2 plus r interest.

Let's start with saving:

The amount of money saved:

$$(m_1 - c_1)$$

In period 2 the consumer gets back:

$$(m_1 - c_1) + r\,(m_1 - c_1)$$

How much can they consume in period 2? They get their income m_2 plus this payment from the bank:

$$c_2 = m_2 + (m_1 - c_1) + r\,(m_1 - c_1)$$

$$c_2 = m_2 + (1 + r)\,(m_1 - c_1)$$

$$(1 + r)\,c_1 + c_2 = (1 + r)\,m_1 + m_2$$

The "price" of consumption in period 1 is $(1 + r)$ and the "price" of consumption in period 2 is 1.

Let's generate the same budget equation by working through an example of borrowing.

Borrow money in period 1. Amount borrowed is:

$$(c_1 - m_1)$$

How much do I pay back in period 2 :

$$(c_1 - m_1) + r\,(c_1 - m_1)$$

What can I consume in period 2. My income minus what I pay the back to cover my loan:

$$c_2 = m_2 - (c_1 - m_1) - r\,(c_1 - m_1)$$

$$c_2 = m_2 - (1 + r)\,(c_1 - m_1)$$

$$c_2 = m_2 - (1 + r)\,c_1 + (1 + r)\,m_1$$

$$(1 + r)\,c_1 + c_2 = (1 + r)\,m_1 + m_2$$

Because the price in period 2 is 1, we are measuring the value of consumption in terms of period 2. **The future value** version of the budget equation.

Divide both sides by $(1 + r)$

$$c_1 + \frac{c_2}{(1 + r)} = m_1 + \frac{m_2}{(1 + r)}$$

The **present value** version of the budget equation.

What can I consume today if I only consume today?

$$c_1 + \frac{0}{(1+r)} = m_1 + \frac{m_2}{(1+r)}$$

$$c_1 = m_1 + \frac{m_2}{(1+r)}$$

This is the c_1 intercept. The c_2 intercept is the future value of income.

$$(1+r)\,c_1 + c_2 = (1+r)\,m_1 + m_2$$

$$c_2 = (1+r)\,m_1 + m_2$$

Suppose utility for consumption is:

$$u\,(c_1, c_2) = c_1 c_2$$

The consumer is going to get $m_1 = m_2 = 1000$
The interest rate is $r = 0.25$

$$(1+r)\,c_1 + c_2 = (1+r)\,m_1 + m_2$$

$$(1.25)\,c_1 + c_2 = (1.25)\,1000 + 1000$$

$$(1.25)\,c_1 + c_2 = 2250$$

Let's find the tangency condition:

$$-\frac{\frac{\partial(c_1 c_2)}{\partial c_1}}{\frac{\partial(c_1 c_2)}{\partial c_2}} = -1.25$$

$$-\frac{c_2}{c_1} = -1.25$$

$$c_2 = 1.25c_1$$

Let's plug this back into the budget equation to get the optimal consumption:

$$(1.25)\,c_1 + c_2 = 2250$$

$$1.25c_1 + 1.25c_1 = 2250$$

$$2.5c_1 = 2250$$

$$c_1 = 900$$

$$c_2 = 1.25\,(900)$$

$$c_2 = 1125$$

This person is a saver since $c_1 < m_1$: $900 < 1000$

The amount they save is 100. In period 2 the bank pays them:

$$100 + (0.25)\,100$$

$$125$$

Income in period 2 is 1000.

Elasticity is a way of measuring changes in a unit-free way by turning everything into percentages.

Three consumers with utility. $m_1 = 10, m_2 = 20, m_3 = 30$

Person 1's utility: $u_1\left(x_1^1, x_1^2\right) = \left(x_1^1\right)\left(x_1^2\right)$

Person 2's utility: $u_2\left(x_2^1, x_2^2\right) = \left(x_2^1\right)\left(x_2^2\right)$

Person 3's utility: $u_3\left(x_3^1, x_3^2\right) = \left(x_3^1\right)\left(x_3^2\right)$

Subscripts refer the person, superscripts refer the the good.

Price for good 1 is p_1 price for good 2 is p_2.

Demand for person i is:

$$\left(x_i^1, x_i^2\right) = \left(\frac{\frac{1}{2}m_i}{p_1}, \frac{\frac{1}{2}m_i}{p_2}\right)$$

Market demand is the sum of individual demands:

$$X^1 = \sum_{i=1}^{3}\left(x_i^1\right)$$

$$X^2 = \sum_{i=1}^{3}\left(x_i^2\right)$$

In this case:

$$X^1 = \frac{\frac{1}{2}(m_1)}{p_1} + \frac{\frac{1}{2}(m_2)}{p_1} + \frac{\frac{1}{2}(m_3)}{p_1}$$

$$X^1 = \frac{\frac{1}{2}(10+20+30)}{p_1} = \frac{30}{p_1}$$

The market demand only depends on the aggregate income but the distribution of income.

13 Class 13- 3/2/2022

Continuing from last class:

Aggregate income $M = \sum_{i=1}^{n} m_i$ (sum of consumer incomes).

In this case:

$$X^1 = \frac{\frac{1}{2}(M)}{p_1}$$

$$X^2 = \frac{\frac{1}{2}(M)}{p_2}$$

In this case, we can use a *representative consumer* (agent) who has the entire aggregate income to figure out what the market demand is.

Any time consumers have the same homothetic preference this will happen.

In terms of the preference relation, $\succsim$ is *homothetic if:*

$$x \succsim x' \Rightarrow tx \succsim tx'$$

$$x \sim x' \Rightarrow tx \sim tx'$$

When preferences are homothetic, indifference curves are parrallel along a ray through the origin. The MRS will remain constant along any ray.

Checking that a utility function represents homothetic preferences.

$$u = x_1 x_2$$

$$MRS = -\frac{x_2}{x_1}$$

$$MRS(1,1) = -\frac{1}{1} = -1$$

$$MRS\left(2,2\right)=-\frac{2}{2}=-1$$

$$MRS\left(t,t\right)=-\frac{t}{t}=-1$$

If the MRS only depends on the ratio of goods, then the preferences are homothetic. Check whether:

$$MRS\left(tx_1,tx_2\right)=MRS\left(x_1,x_2\right)$$

Example of perfect substitutes:

$$u=2x_1+x_2$$

$$MRS\left(x_1,x_2\right)=-\frac{2}{1}=-2$$

$$MRS\left(tx_1,tx_2\right)=-2$$

These are homothetic.

Example of quasi-linear:

$$u=ln\left(x_1\right)+x_2$$

$$MRS\left(x_1,x_2\right)=-\frac{\frac{1}{x_1}}{1}=-\frac{1}{x_1}$$

$$MRS\left(tx_1,tx_2\right)=-\frac{1}{tx_1}$$

$p_1=1, p_2=1$. Tangency condition:

$$-\frac{1}{x_1}=-1$$

$$x_1=1$$

Suppose demand is x_1x_2 for all consumers. Their incomes are $m_1=10$,.... Solve the market demand.

Elasticity is about studying how demand changes in percentage terms.

Price elasticity of demand. approximate interpretation: What is the % change in demand, when price increases by 1%.

Demand can be **elastic**: demand decreases by more than 1% when price increases by 1%.

Demand can be **inelastic**: demand decreases by less than 1% when price increases by 1%.

Demand can be **unit-elastic**: demand decreases by exactly 1% when price increases by 1%.

Price elasticity of demand:

$$\epsilon_{i,i} = \frac{\partial x_i}{\partial p_i}\frac{p_i}{x_i}$$

Cross Price elasticity of demand:

$$\epsilon_{i,j} = \frac{\partial x_i}{\partial p_j}\frac{p_j}{x_i}$$

Income elasticity of demand:

$$\eta_i = \frac{\partial x_i}{\partial m}\frac{m}{x_i}$$

Cobb-douglass demand: $u = x_1 x_2$

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

$$x_2 = \frac{\frac{1}{2}m}{p_2}$$

Price elasticity of good 1:

$$\epsilon_{1,1} = \frac{\partial x_1}{\partial p_1}\frac{p_1}{x_1} = \frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial p_1}\frac{p_1}{\frac{\frac{1}{2}m}{p_1}}$$

$$= -\frac{m}{2p_1^2}\frac{p_1}{\frac{\frac{1}{2}m}{p_1}}$$

$$= -\frac{2p_1}{2p_1} = -1$$

Cross price elasticity of good 1 with resepct to price 2 is zero.

Income Elasticity:

$$\eta_1 = \frac{\partial x_1}{\partial m}\frac{m}{x_1} = \frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial m}\frac{m}{\frac{\frac{1}{2}m}{p_1}}$$

$$= \frac{p_1}{p_1} = 1$$

Perfect Complements: $u = min\{x_1, x_2\}$

$$x_1 = \frac{m}{(p_1 + p_2)}, x_2 = \frac{m}{(p_1 + p_2)}$$

$$\eta_1 = \frac{\partial\left(\frac{m}{(p_1+p_2)}\right)}{\partial m}\frac{m}{\frac{m}{(p_1+p_2)}}$$

$$= 1$$

Cross price elasticity of x_1 with respect to p_2.

$$\epsilon_{1,2} = \frac{\partial\left(\frac{m}{(p_1+p_2)}\right)}{\partial p_2}\frac{p_2}{\frac{m}{(p_1+p_2)}}$$

$$-\frac{m}{(p_1 + p_2)^2}\frac{p_2}{\frac{m}{(p_1+p_2)}}$$

$$= -\frac{p_2}{(p_1 + p_2)}$$

If $p_1 = p_2 = 1$

$$\epsilon_{1,2} = -\frac{1}{2}$$

14 Class 15- 3/16/2022 (Review Problem)

A consumer has utility:$u = \sqrt{x_1} + x_2$. Income m and prices are p_1, p_2.
A) Write down this consumer's budget constraint?

$$\boxed{p_1x_1 + p_2x_2 = m}$$

B) What is the slope of this consumer's budget line?

$$\boxed{-\frac{p_1}{p_2}}$$

C) Write down a condition that implies the slope of the consumer's indifference curve at a bundle is equal to the slope of the budget constraint.

$$\frac{\partial\left(\sqrt{x_1} + x_2\right)}{\partial(x_1)} = \frac{1}{2\sqrt{x_1}}$$

$$\frac{\partial\left(\sqrt{x_1}+x_2\right)}{\partial\left(x_2\right)} = 1$$

$$MRS = -\frac{\frac{1}{2\sqrt{x_1}}}{1}$$

$$-\frac{\frac{1}{2\sqrt{x_1}}}{1} = -\frac{p_1}{p_2}$$

$$\boxed{-\frac{1}{2\sqrt{x_1}} = -\frac{p_1}{p_2}}$$

D) Solve for the consumer's optimal level of x_1. *Assume the consumer has enough income to afford positive levels of both goods at the optimum.*

$$-\frac{1}{2\sqrt{x_1}} = -\frac{p_1}{p_2}$$

$$\boxed{x_1 = \frac{1}{4}\left(\frac{p_2}{p_1}\right)^2}$$

E) Solve for the consumer's optimal level of x_2.

$$p_1x_1 + p_2x_2 = m$$

$$\boxed{x_2 = \frac{m - \frac{1}{4}\frac{p_2^2}{p_1}}{p_2}}$$

F) Is x_1 are normal or inferior for this consumer (or neither)? What about x_2?

x_1 is neither. x_2 is normal.

G) Suppose the $m = 10$, $p_1 = 1$, $p_2 = 2$. What is the consumer's demand?

$$x_1 = \frac{1}{4}\left(\frac{p_2}{p_1}\right)^2$$

$$\boxed{\begin{gathered} x_2 = \frac{10-1}{2} = \frac{9}{2} \\ x_1 = 1 \end{gathered}}$$

H) Suppose the price of p_1 increases to 2. What is the new demand? What is the total change in demand for good 1.

$$x_1 = \frac{1}{4}$$

$$x_2 = \frac{19}{4}$$

Total change in x_1 is $\frac{3}{4}$

I) What income would the consumer need at the new prices to afford the bundle purchased at the old prices?

The old bundle is $\left(1, \frac{9}{2}\right)$. The cost of this at the new prices is:

$$2\left(1\right) + 2\left(\frac{9}{2}\right) = 11$$

Needs \$11 to afford the old bundle.

J) Of the change in demand for good 1 after the price change, how much is due to the the substitution effect and how much is due to the income effect?

To solve for substitution, calculate the demand at the new prices with the income from part I.

$$x_1 = \frac{1}{4}\left(\frac{2}{2}\right)^2 = \frac{1}{4}$$

$$x_2 = \frac{11 - \frac{1}{4}\frac{2^2}{2}}{2} = \frac{21}{4}$$

Old bundle: $\left(1, \frac{9}{2}\right)$. After the price change: $\left(\frac{1}{4}, \frac{19}{4}\right)$. With compensated income: $\left(\frac{1}{4}, \frac{21}{4}\right)$.

The total effect was $\frac{3}{4}$

$1 - \frac{1}{4} = \frac{3}{4}$ is due to substitution.
0 is due to the income effect.

15 Class 16- 3/28/2022

Production Functions

Inputs x_1, x_2. Output y

Production Function: $f\left(x_1, x_2\right)$

Production function for pies. 2 apples, 1 crust to make a pie.

$f\left(2,1\right) = 1, f\left(3,1\right) = 1, f\left(4,2\right) = 2, f\left(6,3\right) = 3$

$f\left(x_1, x_2\right) = min\left\{\frac{1}{2}x_1, x_2\right\}$

$g\left(x_1, x_2\right) = 2\left(min\left\{\frac{1}{2}x_1, x_2\right\}\right)$

Isoquants

Isoquant is analogous to the indifference curve.

Iso- same. Quant- quantity.

Bundles of input that give the same quantity of output.

Example - Fixed Proportions/Perfect Complements

Need a units of x_1 and b units of x_2 to make each unit of output.

$f\left(x_1, x_2\right) = min\left\{\frac{1}{a}x_1, \frac{1}{b}x_2\right\}$

Example - Perfect Substitutes

$f\left(x_1, x_2\right) = ax_1 + bx_2$

$f\left(x_1, x_2\right) = x_1 + 2x_2$

Technical Rate of Substitution

The technical rate of substitution (**TRS**) is the slope of an isoquant and measures

How much x_2 can I remove, if I add one unit of x_1 in order to produce the same amount.

$$-\frac{\frac{\partial(f(x_1,x_2))}{\partial x_1}}{\frac{\partial(f(x_1,x_2))}{\partial x_2}}$$

The TRS represents the trade-offs inherent in a particular technology.

$$-\frac{\frac{\partial(x_1+2x_2)}{\partial x_1}}{\frac{\partial(x_1+2x_2)}{\partial x_2}} = -\frac{1}{2}$$

A family of production functions called the **CES.** This is an example:

$$f\left(x_1, x_2\right) = \left(x_1 + x_2\right)^{\frac{1}{2}}$$

$$TRS = -1$$

$$f\left(x_1, x_2\right) = x_1 + x_2$$

$$TRS = -1$$

Returns to Scale

How does production change when we increase both inputs at the same time.

$$f\left(x_1, x_2\right) = x_1 + x_2$$

$$f\left(x_1, x_2\right) = 2x_1 + 2x_2 = 2\left(x_1 + x_2\right)$$

$$f\left(2x_1, 2x_2\right) = 2f\left(x_1, x_2\right)$$

Doubling both inputs doubles the output.

$$f\left(tx_1, tx_2\right) = tf\left(x_1, x_2\right)$$

Multiplying input by t multiplies output by t.

Constant (linear) returns to scale.

$$f\left(2x_1, 2x_2\right) = \left(2x_1 + 2x_2\right)^{\frac{1}{2}} = \left(2\left(x_1 + x_2\right)\right)^{\frac{1}{2}} = \sqrt{2}\left(x_1 + x_2\right)^{\frac{1}{2}}$$

$$f\left(2x_1, 2x_1\right) = 1.41421 f\left(x_1, x_2\right)$$

Decreasing returns to scale.

For any $t > 1$

$$f\left(tx_1, tx_2\right) < tf\left(x_1, x_2\right)$$

$f\left(x_1, x_2\right) = \left(x_1 + x_2\right)^2$

$$a\left(x_1 + x_2\right)^2$$

$$f\left(2x_1, 2x_2\right) = \left(2x_1 + 2x_2\right)^2 = \left(2\left(x_1 + x_2\right)\right)^2 = 4\left(x_1 + x_2\right)^2$$

Double inputs, quadruples output.

Increasing Returns to Scale.

$$f\left(tx_1, tx_2\right) > tf\left(x_1, x_2\right)$$

Marginal Product

This is analogous to marginal utility. How does production increase as I increase **one of the inputs** a little bit.

$MP_1 = \frac{\partial(f(x_1,x_2))}{\partial x_1}$ how much will production increase if I increase x_1?

$MP_2 = \frac{\partial(f(x_1,x_2))}{\partial x_2}$ how much will production increase if I increase x_2?

$$TRS = -\frac{MP_1}{MP_2}$$

Diminishing Marginal Product.

Each input becomes less and less productive as you increase input while holding the others fixed. Diminishing marginal product requires that the second derivatives of the production function are negative.

$$\frac{\partial\left(f\left(x_1,x_2\right)\right)}{\partial\left(x_1\right)^2}<0$$

Examples:

$$f\left(x_1,x_2\right)=x_1^{\frac{1}{3}}x_2^{\frac{1}{3}}$$

What are the returns to scale?

$$f\left(tx_1,tx_2\right)=\left(tx_1\right)^{\frac{1}{3}}\left(tx_2\right)^{\frac{1}{3}}=t^{\frac{1}{3}}t^{\frac{1}{3}}\left(x_1\right)^{\frac{1}{3}}\left(x_2\right)^{\frac{1}{3}}=t^{\frac{2}{3}}x_1^{\frac{1}{3}}x_2^{\frac{1}{3}}$$

$$=t^{\frac{2}{3}}f\left(x_1,x_2\right)$$

For any $t>1$ $t^{\frac{2}{3}}<t$
For any $t>1$

$$f\left(tx_1,tx_2\right)<tf\left(x_1,x_2\right)$$

This has decreasing returns to scale.

Does it have diminishing marginal product?

$$MP_1=\frac{\partial\left(x_1^{\frac{1}{3}}x_2^{\frac{1}{3}}\right)}{\partial x_1}=\frac{1}{3}x_1^{-\frac{2}{3}}x_2^{\frac{1}{3}}=\frac{1}{3}\frac{x_2^{\frac{1}{3}}}{x_1^{\frac{2}{3}}}$$

$$-\frac{2\sqrt[3]{x_2}}{9x_1^{5/3}}<0$$

$$MP_2=MP_1=\frac{\partial\left(x_1^{\frac{1}{3}}x_2^{\frac{1}{3}}\right)}{\partial x_2}=\frac{1}{3}x_2^{-\frac{2}{3}}x_1^{\frac{1}{3}}$$

Example $x_1^{\alpha}x_2^{\beta}$

$$f\left(tx_1,tx_2\right)=\left(tx_1\right)^{\alpha}\left(tx_2\right)^{\beta}=t^{\alpha}t^{\beta}x_1^{\alpha}x_2^{\beta}=t^{\alpha+\beta}x_1^{\alpha}x_2^{\beta}$$

for a $t>1$, $t^{\alpha+\beta}>t$ if and only if $\alpha+\beta>1$
for a $t>1$, $t^{\alpha+\beta}<t$ if and only if $\alpha+\beta<1$
Cobb Douglass has increasing returns if and only if the sum of the exponents is strictly greater than 1

Cobb Douglass has decreasing returns if and only if the sum of the exponents is strictly less than 1

Cobb Douglass has constant returns if and only if the sum of the exponents is equal 1

$$x_1^{\frac{2}{3}} x_2^{\frac{2}{3}}$$

Increasing returns to scale. For instance, if we double input, we get 2.51984 times the amount of output.

$$MP_1 = \frac{\partial \left(x_1^{\frac{2}{3}} x_2^{\frac{2}{3}} \right)}{\partial x_1} = \frac{2x_2^{\frac{2}{3}}}{3x_1^{\frac{1}{3}}}$$

Diminishing marginal product.

$$x_1^{\frac{1}{\ }} x_2^2$$

For Cobb Douglass, an input has diminishing marginal product if and only if its exponent is in the interval $(0, 1)$.

$$x_1^2 x_2^2$$

16 Class 17- 3/30/2022

Profit Maximization

The price of output is p. (Price taking assumption. The firms are operating in perfect competition.)

The prices (wages) for the inputs are given by w_1, w_2.

The objective of a firm is to maximize profit.

Revenue is:

$$pf(x_1, x_2)$$

Cost is:

$$x_1 w_1 + x_2 w_2$$

Profit is revenue minus cost:

$$\pi(x_1, x_2) = pf(x_1, x_2) - (x_1 w_1 + x_2 w_2)$$

Necessary Condition for Profit Maximization:

$$\frac{\partial\left(\pi\left(x_1,x_2\right)\right)}{\partial x_1}=0$$

$$\frac{\partial\left(\pi\left(x_1,x_2\right)\right)}{\partial x_2}=0$$

Suppose it wasn't the case. Suppose:

$$\frac{\partial\left(\pi\left(x_1,x_2\right)\right)}{\partial x_1}>0$$

$$\frac{\partial\left(\pi\left(x_1,x_2\right)\right)}{\partial x_2}>0$$

Then increasing either x_1 or x_2 will increase profit.

$$\frac{\partial\left(\pi\left(x_1,x_2\right)\right)}{\partial x_1}<0$$

$$\frac{\partial\left(\pi\left(x_1,x_2\right)\right)}{\partial x_2}<0$$

Then decreasing either x_1 or x_2 will increase profit.

$$Rev-Cost$$

$$\frac{\partial\left(\pi\right)}{x_1}=\frac{\partial\left(Rev\right)}{x_1}-\frac{\partial\left(Cost\right)}{x_1}$$

$$\frac{\partial\left(\pi\right)}{x_1}=0$$

This gives us the fundamental profit maximization condition:

$$MR_1=MC_1$$

$$MR_2=MC_2$$

Example. A Cobb Douglass production problem:
$p=2$,$w_1=1,w_2=1$. $f\left(x_1,x_2\right)=x_1^{\frac{1}{3}}x_2^{\frac{1}{3}}$

$$\pi\left(x_1,x_2\right)=2\left(x_1^{\frac{1}{3}}x_2^{\frac{1}{3}}\right)-x_1-x_2$$

Necessary conditions for profit max (first-order conditions). First we find the marginal profits.

$$\frac{\partial\left(2\left(x_1^{\frac{1}{3}}x_2^{\frac{1}{3}}\right)-x_1-x_2\right)}{\partial x_1} = 2\left(\frac{1\sqrt[3]{x_2}}{3x_1^{2/3}}\right)-1$$

$$\frac{\partial\left(2\left(x_1^{\frac{1}{3}}x_2^{\frac{1}{3}}\right)-x_1-x_2\right)}{\partial x_2} = \frac{2\sqrt[3]{x_1}}{3x_2^{2/3}}-1$$

Set these to zero and solve:

$$\frac{2\sqrt[3]{x_2}}{3x_1^{2/3}}=1$$

$$\frac{2\sqrt[3]{x_1}}{3x_2^{2/3}}=1$$

$$Solve[\{\frac{2\sqrt[3]{x_2}}{3x_1^{2/3}}==1,\frac{2\sqrt[3]{x_1}}{3x_2^{2/3}}==1\},\{x_1,x_2\}]$$

$$x_1=\frac{8}{27},x_2=\frac{8}{27}$$

$$y=0.444444$$

Example. A Cobb Douglass production problem:
$p=1$,$w_1=1,w_2=1$. $f(x_1,x_2)=x_1^{\frac{1}{2}}x_2^{\frac{1}{2}}$

$$\pi(x_1,x_2)=\left(x_1^{\frac{1}{2}}x_2^{\frac{1}{2}}\right)-x_1-x_2$$

$$\frac{\partial\left(\left(x_1^{\frac{1}{2}}x_2^{\frac{1}{2}}\right)-x_1-x_2\right)}{\partial x_1} = \frac{10\sqrt{x_2}}{2\sqrt{x_1}}-1=0$$

$$\frac{\partial\left(\left(x_1^{\frac{1}{2}}x_2^{\frac{1}{2}}\right)-x_1-x_2\right)}{\partial x_2} = \frac{10\sqrt{x_1}}{2\sqrt{x_2}}-1=0$$

$$(0,0)$$

17 Class 18- 4/4/2022

Cost Minimization:

$$p(x_1, x_2) - w_1x_1 - w_2x_2$$

y, cheapest bundle of x_1, x_2 to produce y. Necessary condition for profit maximization.

$Min_{x_1,x_2}(w_1x_1 + w_2x_2)$ subject to $f(x_1, x_2) = y$

Isocost curves. $w_1x_1 + w_2x_2 = c$ isocost curve for cost c

$w_1x_1 + w_2x_2 = 10$

$w_1x_1 + w_2x_2 = 20$

This problem is **dual** to a similar problem we have seen: utility maximization. Minimization requires a similar condition.

Slope of the isoquant = slope of the isocost at minimizing bundle.

Find the minimum cost bundle of inputs for producing y output with production function $f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$. $w_1 = 1$, $w_2 = 1$

$$-\frac{\frac{\partial\left(x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}\right)}{\partial x_1}}{\frac{\partial\left(x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}\right)}{\partial x_2}} = -1$$

$$-\frac{x_2}{x_1} = -1$$

$$x_1 = x_2$$

Where cost minimization procedure differs, we need plug this back into the production constraint.

$$x_1^{\frac{1}{2}} x_1^{\frac{1}{2}} = y$$

$$x_1^* = y$$

$$x_2^* = y$$

These are the **conditional factor demands.**

If we plug these back into the cost equation $c(y) = w_1x_1^* + w_2x_2^*$ we get the **cost function**.

The cost function tells us the cost of the cheapest bundle we could possibly use to produce output y.

We can write a new profit function that is only a function of y (and the price of output).

$$\pi(y) = py - c(y)$$

In the problem above if $w_1 = 1$ and $w_2 = 1$

$$c(y) = y + y = 2y$$

If the price of output is p, the profit function in terms only of y is:

$$\pi(y) = py - 2y = (p-2)\,y$$

Another example. A Cobb Douglass production problem:

$p = 2$,$w_1 = 1, w_2 = 1$. $f(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{1}{3}}$

TRS=$-\frac{w_1}{w_2}$

$$-\frac{x_2}{x_1} = -1$$

$$x_2 = x_1$$

Plug this into the production constraint:

$$x_1^{\frac{1}{3}} x_1^{\frac{1}{3}} = y$$

$$x_1^{\frac{2}{3}} = y$$

Conditional factor demands:

$$x_1 = y^{\frac{3}{2}}$$

$$x_2 = y^{\frac{3}{2}}$$

Cost function. $w_1 = w_2 = 1$

$$c(y) = y^{\frac{3}{2}} + y^{\frac{3}{2}} = 2y^{\frac{3}{2}}$$

Profit function it terms of y. Recall that $p = 2$

$$\pi(y) = 2y - 2y^{\frac{3}{2}}$$

To maximize this, it is necessary that the slope be zero. The marginal profit with respect to output y has to be zero.

$$\frac{\partial\left(2y-2y^{\frac{3}{2}}\right)}{\partial y} = 2-3\sqrt{y}=0$$

$$\frac{2}{3}=\sqrt{y}$$

$$0.444444=y$$

$$x_2^*=0.296296$$

$$x_1^*=0.296296$$

How much can this firm earn?

$$\pi^*=0.296296$$

18 Class 19- 4/6/2022

$$c\left(y\right)=\left(y^2+y\right)+\left(100\right)$$

We can decompose a cost function into two parts, **variable cost** and the **fixed cost**. Variable cost changes with production, fixed cost is fixed. $c\left(0\right)$

In this case:

$$vc\left(y\right)=y^2+y$$

$$fc=100$$

$$mc\left(y\right)=2y+1$$

18.1 Why is inverse supply equal to the marginal cost function?

For any firm, profit maximization occurs where:

$$MR\left(y\right) = MC\left(y\right)$$

$$\pi\left(y\right) = Rev\left(y\right) - c\left(y\right)$$

$$p\left(y\right) y - c\left(y\right)$$

$$MR = MC$$

$$p'\left(y\right) y + p\left(y\right) = mc\left(y\right)$$

For a price-taking firm, $p'\left(y\right) = 0$ because they assume price is fixed in their output. The firm produces where $mc\left(y\right) = p$

$$py - c\left(y\right)$$

$$p = mc\left(y\right)$$

Returning to our example from last class, let's suppose that the firm wants to determine how much to supply any **any** price:

$$\pi\left(y\right) = py - 2y^{\frac{3}{2}}$$

$$mc\left(y\right) = 3\sqrt{y}$$

At the optimum, they want to find the output y^{*} that sets the $mc\left(y^{*}\right) = p$

$$p = 3\sqrt{y}$$

This is our supply function.

$$y = \left(\frac{p}{3}\right)^{2}$$

At what price will the firm produce 100 units of output?

$$100 = \left(\frac{p}{3}\right)^{2}$$

$$30 = p$$

Inverse supply:

$$y = \left(\frac{p}{3}\right)^2$$

$$p = 3y^{\frac{1}{2}}$$

18.2 Profit

$$\pi(y) = py - 2y^{\frac{3}{2}}$$

$p = 30$

$$\pi = 30(100) - 2(100)^{\frac{3}{2}}$$

$$\pi = 1000$$

Optimized profit function:

$$\pi(p) = \frac{1}{9}p^3 - \frac{2}{27}p^3 = \frac{1}{27}p^3$$

Let's suppose the firm had a fixed cost of 1000

$$\pi(y) = py - 2y^{\frac{3}{2}} - 1000$$

The firm's behavior should be exactly the same. Firm always ignore fixed costs in choosing output.

$$\pi(p) = \frac{1}{27}p^3 - 1000$$

$$\pi(30) = 0$$

$$\pi(10) = -962.963$$

Profit of producing zero

$$= -1000$$

18.3 $y = 0$

$$c(y) = y^2 + 10y + 100$$

Find the optimal y for any p. Find the supply function.

$$\pi(y) = py - \left(y^2 + 10y + 100\right)$$

$$p = \frac{\partial\left(\left(y^2 + 10y + 100\right)\right)}{\partial y}$$

$$p = 2y + 10$$

$$y = \frac{p - 10}{2}$$

They want to produce $y = \frac{p-10}{2}$ if $p \geq 10$

18.4 Example

Suppose $c(y) = 4y^2 + 50$.
This firm is a price taker. Find the firms supply function:

$$p = 8y$$

What would the price need to be for this firm to product 10 units of output?

$$p = 8(10) = 80$$

The supply function is:

$$y = \frac{1}{8}p$$

What would the price need to be so this firm earns positive profit?

$$p\left(\frac{1}{8}p\right) - 4\left(\left(\frac{1}{8}p\right)^2\right) - 50$$

$$\frac{2}{16}p^2 - \frac{1}{16}p^2 - 50$$

$$= \frac{1}{16}p^2 - 50$$

$$\frac{1}{16}p^2 = 50$$

$$p^2 = 800$$

$$p = \sqrt{800}$$

$$p = 28.2843$$

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$p = mc\,(y)$ is necessary for a profit max **with** $y > 0$**.**
Two things that can go wrong:
First. Any y that solves $p = mc\,(y)$ is negative. The optimal output is $y = 0$.
Second. Any $y > 0$ that solves problem gives a profit less than $\pi\,(0)$. That is, less than producing nothing.

19.1 Example

Suppose we have $c\,(y) = y\,(y-10)^2 + 10y$. Suppose $p = 8$. Profit is:

$$\pi\,(y) = (8)\,y - \left(y\,(y-10)^2 + 10y\right)$$

$$8 = (y-10)^2 + 2y(y-10) + 10$$

$$\left\{\left\{y \to \frac{1}{3}\left(20 - \sqrt{94}\right)\right\}, \left\{y \to \frac{1}{3}\left(20 + \sqrt{94.0}\right)\right\}\right\}$$

$$3.43488$$

$$9.89845$$

The second point is a local max, let's check the profit:

$$\pi\,(y) = -19.899$$

This is less than producing $y = 0$, so it is not the optimal output.

$$\pi\,(0) = 0$$

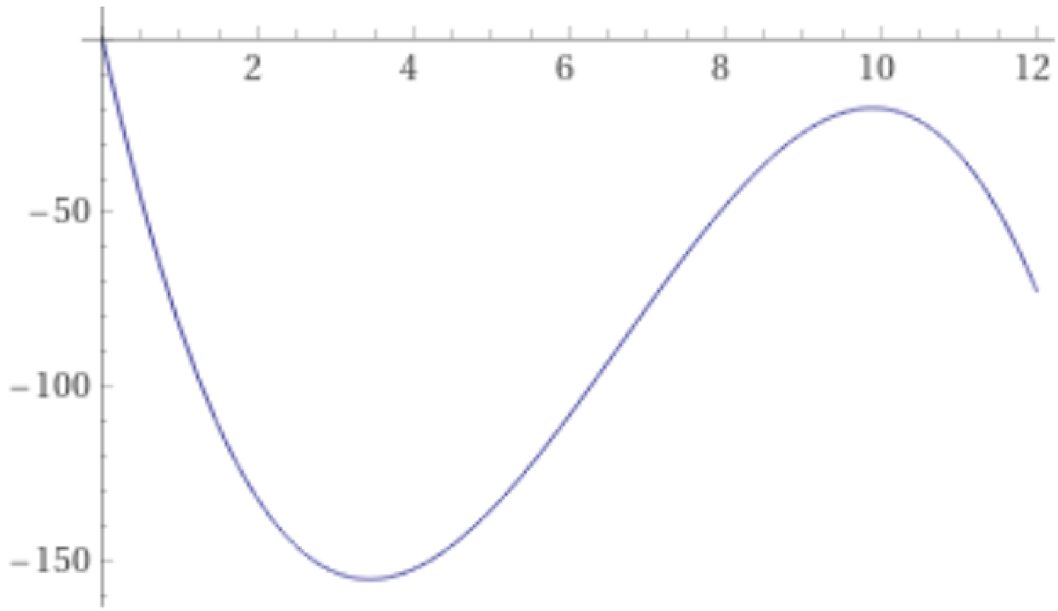

Figure 2: A plot of $\pi(y) = 8y - y(y-10)^2 - 10y$

19.2 Industry Supply

all the firms in the industry have $c(y) = y^2$

$$\pi(y) = py - y^2$$

$$p = 2y$$

$$y = \frac{p}{2}$$

19.3 Summing Supply Functions

Individual firm i supply is $y_j = \frac{p}{2}$.

Suppose there are J firms. The market supply, $Y(p) = \sum_{j=1}^{J} y_j(p)$.

This is the supply function for the example above.

$$Y(p) = \sum_{j=1}^{J} \frac{p}{2} = J\frac{p}{2}$$

Inverse supply:

$$Y = J\frac{p}{2}$$

$$p = 2\frac{Y}{J}$$

In perfectly competitive markets (all the firms are price takers) an increase in the number of firms will "flatten" the inverse supply function: the elasticity of supply will increase.

19.4 "Flattening" The Industry Supply

If we have J firms with the supply function $y = \frac{p}{2}$:

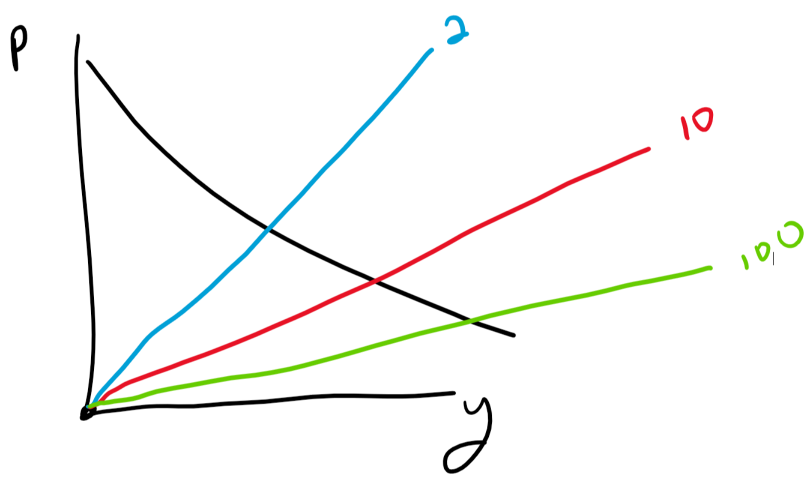

Figure 3: The industry supply "flattens" as more firms enter the market.

Figure 4: Industry supply as the number of firms increase when the minimum of average variable cost is above zero.

19.5 Different Supply Functions

It two firms had different supply functions:

$$y_1(p) = \frac{p}{2}, y_2 = p$$

$$Y = y_1(p) + y_2(p)$$

$$Y = \frac{p}{2} + p = \frac{3}{2}p$$

19.6 Monopolies and the Price-Taking Assumption

A single firm serving a market **cannot reasonably** assume that price is fixed in their output.

For a monopolist, the most they charge for output y is the most that consumers will pay for it, that is exactly the inverse demand at y. Instead of py, the revenue function is now $p(y)y$ where $p(y)$ is the inverse demand.

$$\pi(y) = p(y)y - c(y)$$

Monopolist has $c(y) = y^2$ and demand is $Q(p) = 100 - p$. The inverse demand is: $p = 100 - y$

$$\pi(y) = ((100 - y)y) - y^2$$

$$\left(100y - y^2\right) - y^2$$

$$\pi\left(y\right) = 100y - 2y^2$$

$$\frac{\partial\left(\pi\left(y\right)\right)}{\partial y} = 100 - 4y$$

$$100 - 4y = 0$$

$$y = 25$$

$$p = 100 - 25 = 75$$

$$\pi = 1250$$

The marginal cost for the monopolist is:

$$mc\left(y\right) = 2y$$

$$mc\left(25\right) = 50$$

$$p = 75$$

There is “markup” of $25 by this monopolist.
Let’s calculate the elasticity of demand for consumers:

$$\frac{\partial\left(100 - p\right)}{\partial p}\frac{p}{100 - p}$$

$$-1\left(\frac{p}{100 - p}\right) = -3$$

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20.1 Elasticity and Markup

$$\pi(y) = p(y)\,y - c(y)$$

First order condition for a monopolist.

$$\left(\frac{\partial p}{\partial y}y + p\right) - mc = 0$$

$$\left(\frac{\partial p}{\partial y}y\right) + p = mc$$

$\varepsilon = \frac{\partial y}{\partial p}\frac{p}{y}$, $\frac{1}{\varepsilon} = \frac{\partial p}{\partial y}\frac{y}{p}$
Divide both sides by p

$$\left(\frac{\partial p}{\partial y}\frac{y}{p}\right) + \frac{p}{p} = \frac{mc}{p}$$

$$\left(\frac{1}{\varepsilon}\right) + 1 = \frac{mc}{p}$$

$$\left(\frac{1+\varepsilon}{\varepsilon}\right) = \frac{mc}{p}$$

$$\frac{p}{mc} = \frac{\varepsilon}{1+\varepsilon}$$

Let's check the example above:

$$\frac{p}{mc} = \frac{-3}{1+(-3)} = \frac{-3}{-2} = \frac{3}{2}$$

Let's say we observe price is 10 and $mc = 1$.

$$\frac{10}{1} = \frac{\varepsilon}{1+\varepsilon}$$

$$10 + 10\varepsilon = \varepsilon$$

$$\varepsilon = -\frac{10}{9}$$

As long as $\varepsilon < -1$, $\frac{\varepsilon}{1+\varepsilon} > 1$.

20.2 Another Example

Monopolist has $c(y) = y^2$ and demand is $Q(p) = 100 - p$.
What is the inverse demand function?

$$y = 100 - p$$

$$p = 100 - y$$

What is the monopolist's profit function in terms only of y?

$$\pi(y) = y(100 - y) - y^2$$

What is the profit maximizing output for this firm?
The first order condition:

$$\frac{\partial\left(y(100-y) - y^2\right)}{\partial y} = 100 - 4y$$

$$100 - 4y = 0$$

$$100 = 4y$$

$$y = 25$$

20.3 Another Example

Demand is $q = \frac{1000}{p^2}$. Elasticity of demand is:

$$\frac{\partial q}{\partial p}\frac{p}{q}$$

$$\frac{\partial\left(\frac{1000}{p^2}\right)}{\partial p}\frac{p}{\frac{1000}{p^2}}$$

$$-\frac{2000}{p^3}\frac{p^3}{1000} = -2$$

Firm's cost function is $c(y) = 10y$. $mc = 10$.
Write down the amount price will be marked up above marginal cost at the optimum for this firm:

$$\frac{\varepsilon}{1+\varepsilon} = \frac{-2}{1+-2} = 2$$

What is the price this firm will charge at the optimum?

$$p = 2 * 10 = 20$$

What is the optimal quantity?

$$y = \frac{5}{2}$$

The "old-fashioned" way. Inverse demand is:

$$p = \left(\frac{1000}{y}\right)^{\frac{1}{2}}$$

Profit function:

$$y^1 \left(\frac{1000}{y}\right)^{\frac{1}{2}} - 10y$$

$$y^1 \frac{1}{y^{\frac{1}{2}}} (1000)^{\frac{1}{2}} - 10y$$

$$(1000)^{\frac{1}{2}} y^{\frac{1}{2}} - 10y$$

First order condition (take derivative and set to 0):

$$\frac{1}{2} (1000)^{\frac{1}{2}} y^{-\frac{1}{2}} = 10$$

$$\frac{1}{2} (1000)^{\frac{1}{2}} \frac{1}{y^{\frac{1}{2}}} = 10$$

$$\frac{1}{10} \frac{1}{2} (1000)^{\frac{1}{2}} = y^{\frac{1}{2}}$$

$$\frac{1000}{400} = y$$

$$y = \frac{5}{2}$$

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3,2,1

Price of 2. Profit of 4.

Profit of selling to all people at their WTP (willingness to pay) profit of 6.

21.1 Types of Price Discrimination

First-Degree Price Discrimination: Charge each person their willingness to pay.

Consumers get 0 surplus, the monopolist captures all of the surplus/welfare available in the market. There is no dead-weight loss because the quantity sold is maximized (same quantity as in perfect competition).

There are two factors here: **1. ability to charge different prices.** 2. ability to identify people who have higher willingness to pay.

Second-Degree Price Discrimination: Offer different quantity/quality options, let consumers **self-select** what they purchase.

Wine: Reserve wines vs "regular" wines.

Internet Service: Gig internet, high-speed, dial-up.

Airlines: First-class/Coach Service.

Gas: different octane levels.

Third-Degree Price Discrimination: *Identify groups of individuals.* Know something about those groups.

Student discounts.

21.2 Third Degree Price Discrimination

Suppose there are two groups of people: students and non-students and a firm that sells some good to both groups. Assume the firm has zero marginal cost.

Students have demand function:

$$y_s = 100 - 2p$$

Non-students have demand function:

$$y_n = 100 - p$$

Total demand in this market is $y_s + y_n = Y$.

$$Y = (100 - 2p) + (100 - p)$$

$$Y = 200 - 3p$$

The inverse demands.
Students: $p = \frac{100-y_s}{2} = 50 - \frac{1}{2}y_s$
Non-students: $p = 100 - y_n$
Total: $p = \frac{200}{3} - \frac{1}{3}y$
What is the optimal quantity and price if there is no student discount?

$$\pi = y\left(\frac{200}{3} - \frac{1}{3}y\right)$$

$$\frac{\partial\left(y\left(\frac{200}{3} - \frac{1}{3}y\right)\right)}{\partial y} = \frac{200}{3} - \frac{2y}{3}$$

Set this to zero to find the optimal y.

$$\frac{200}{3} = \frac{2y}{3}$$

$$y = 100$$

The optimal price is (plug this into the market inverse demand):

$$p = \left(\frac{200}{3} - \frac{1}{3}100\right) = 33.3333$$

$$\pi = 3333.33$$

$$y_s = 33.3333$$

$$y_n = 66.6667$$

If we charge different prices, we can solve the optimal price and quantity treating each group as a separate market.

$$\pi_n = y_n\left(100 - y_n\right)$$

$$\frac{\partial\left(y_n\left(100 - y_n\right)\right)}{\partial y_n} = 100 - 2y_n$$

$$y_n = 50$$

$$p_n = 50$$

$$\pi_n = 2500$$

$$\pi_s = y_s \left(50 - \frac{1}{2} y_s\right)$$

$$\frac{\partial \left(y_s \left(50 - \frac{1}{2} y_s\right)\right)}{\partial y_s} = 50 - y_s$$

$$y_s = 50$$

$$p_s = 25$$

$$\pi_s = 1250$$

Total profit:

$$\pi_{s+n} = 3750$$

21.3 Bundling

This can occur when a company sells different types of products. They can force consumers to buy bundles instead of separate products and earn more money.

Willingness to pay:

	Shirt	Pants	Both
Consumer 1	50	30	80
Consumer 2	10	80	90

Let's suppose they allow consumers to buy shirt or a pair of pants or both.

Shirts:

Sell to both consumers: Most I can charge is \$10. Profit is 2*10=\$20

Sell to one consumer: \$50. Profit is \$50.

Optimal to price at \$50 and sell 1 shirt.

Pants:

Both: Charge \$30. Profit: \$60.

One: Charge \$80. Profit: \$80.

Optimal to price at \$80 and sell 1 pair of pants.

Total profit from allowing these to be bought separately is \$130.

Bundles:

Both: charge \$80. Profit: \$160

One: charge \$90. Profit: \$90

Optimal to sell bundle to both at a price of \$80.

21.4 Two-Part Tariff

This will be possible when consumers demand more than one unit of the product/service you sell.

Two parts: 1. a low unit-cost. 2. a high up-front cost.

Suppose a consumer's demand for coffee is $q = 10 - p$ and the firm has zero marginal cost for coffee.

$$\pi = q\,(q - 10)$$

$$q^* = 5$$

$$p = 5$$

$$25$$

If the firm sells to that consumer at a single price it's profit of selling the consumer q cups of coffee at the most they will pay for those q cups is:

$$\pi = (10 - q)\,q$$

$$\$50$$

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22.1 Two Part Tariff

Monthly demand for a single consumer:

$q = (10 - p)$

Inverse demand:

$p = (10 - q)$

Cost for the firm is 0 per cup of coffee.

Charging a single price. Here is the optimum:

$$\pi = (10 - q)(q) = 10q - q^2$$

$$q^* = 5$$

$$p^* = 5$$

$$\pi = 25$$

If the monopolist offers \$50 up-front fee, for the right to coffee. The consumer will indifferent. Profit is \$50.

As a rule of thumb, the unit cost under two part tariff is $p = mc$. The "entry fee" is equal to the consumer welfare (area below the inverse demand and above price).

Let's suppose $mc = 1$. Unit price is \$1. Consumer welfare under this pricing is 40.5. This is the upfront fee.

22.2 A Bridge Between Monopoly and Perfect Competition

Cournot Oligopoly.

Number of firms: N

Firm i's quantity is: q_i

Total quantity is: $Q = \sum_{i=1}^{N} q_i$

Total quantity of all firms except i: $Q_{-i} = Q - q_i = \sum_{i \in \{1,\ldots,N\}/\{i\}} q_i$

$N = 3$. $q_1 = 5$,$q_2 = 10, q_3 = 15$.

$Q = 5 + 10 + 15 = 30$

$Q_{-1} = 10 + 15 = 25$

$Q_{-2} = 5 + 15 = 20$

$Q_{-3} = 5 + 10 = 15$

What is a firm's profit in this market?

Revenue of firm i. Let $p(Q)$ be the inverse demand.

$$(p(Q_{-i} + q_i))\, q_i$$

Cost of firm i (in our class we will assume all firms have the same cost function):

$$c(q_i)$$

Profit function of firm i.

$$\pi_i = p\left(Q_{-i} + q_i\right) q_i - c\left(q_i\right)$$

Example.

2 firms have cost $c\left(q_i\right) = 10q_i$ and demand is $q = 100 - p$.

Inverse demand $p = 100 - Q$.

Set up each firm's profit function:

$$\pi_1 = \left(100 - \left(q_2 + q_1\right)\right) q_1 - 10q_1$$

$$100q_1 - q_2q_1 - q_1^2 - 10q_1$$

$$\frac{\partial\left(\left(100 - \left(q_2 + q_1\right)\right) q_1 - 10q_1\right)}{\partial q_1} = 100 - q_2 - 2q_1 - 10$$

$$100 - q_2 - 2q_1 - 10 = 0$$

$$q_1 = 45 - \frac{1}{2}\left(q_2\right)$$

This is what we call firm 1's "Best Response Function". This gives us the optimal q_1 for any quantity q_2 chosen by the other firm.

$$\pi_2 = \left(100 - \left(q_1 + q_2\right)\right) q_2 - 10q_2$$

$$\frac{\partial\left(\left(100 - \left(q_1 + q_2\right)\right) q_2 - 10q_2\right)}{\partial q_2} = -q_1 - 2q_2 + 90$$

$$-q_1 - 2q_2 + 90 = 0$$

Firm 2's best response:

$$q_2 = 45 - \frac{1}{2}q_1$$

Let's suppose $q_2 = 10$. The best response of firm 1 would be:

$$q_1 = 45 - \frac{1}{2}(10) = 40$$

So firm 1 will produce 40. What is firm 2's best response? Is it 10?

$$q_2 = 45 - 20 = 25$$

$(40, 10)$ is not a Nash equilibrium because $q_1 = 40$ is a best response to $q_2 = 10$, $q_2 = 10$ is not a best response to $q_1 = 40$.

A **Nash Equilibrium** is a set of quantities for all firms such that each firm is best responding the quantities of the other firms.

q_1 is a best response to q_2 and q_2 is a best response to q_1. Solve the system of equations:

$$q_1 = 30, q_2 = 30$$

What is firm 2's best response to 30?

$$45 - \frac{1}{2}(30) = 30$$

In this this case, firm 1 doesn't want to change what their doing, given what firm 2 is doing and the same for firm 2.

Market quantity $Q = 30 + 30 = 60$. $p = 100 - 60 = 40$

$$\pi_i = (40 * 30) - 10(30) = 900$$

For comparison, what would a monopolist do?

$$\pi = (100 - q)q - 10q$$

$$\frac{\partial((100 - q)q - 10q)}{\partial q} = 90 - 2q$$

$$q = 45$$

$$p = 55$$

$$\pi = 2025$$

For every firm that enters this market, quantity will go up and price will go down. The total producer welfare (total profit of the firms) will decrease.

Let's now assume there are N firms.

$$\pi_i = (100 - (Q))q_i - 10q_i$$

$$\pi_i = (100 - (Q_{-i} + q_i))\, q_i - 10q_i$$

$$\frac{\partial\left((100 - (Q_{-i} + q_i))\, q_i - 10q_i\right)}{\partial q_i} = 100 - Q_{-i} - 2q_i - 10$$

$$q_i = 45 - \frac{1}{2}Q_{-i}$$

Any equilibrium will be symmetric $q_i = q_j$ for all i and j. There are $N-1$ firms that aren't i. Impose symmetry by setting $q_i = q_j = q^*$.

$$q^* = 45 - \frac{1}{2}\left((N-1)\, q^*\right)$$

$$q^* + \frac{1}{2}\left((N-1)\, q^*\right) = 45$$

$$2q^* + (N-1)\, q^* = 90$$

$$(N+1)\, q^* = 90$$

$$q^* = \frac{90}{N+1}$$

For $N = 2$

$$q^* = \frac{90}{2+1} = 30$$

Market quantity:

$$Q = N\,(q^*) = \frac{N}{N+1}90$$

As $N \to \infty$, market quantity approaches 90. Price approaches $100 - 90 = 10$.

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23.1 One Last Example

Suppose demand is $q(p) = 500 - p$. Any firm's cost function is $c(q) = 50q + 100$.
A) What is the inverse demand function?

$$q(p) = 500 - p$$

$$p = 500 - q$$

B) At what price would there be 400 units of demand?

$$p = 500 - 400 = 100$$

C) Suppose firms are price takers, the market price is 100. Set up the firm's profit functions.

$$\pi = 100q - 50q - 100 = 50q - 100$$

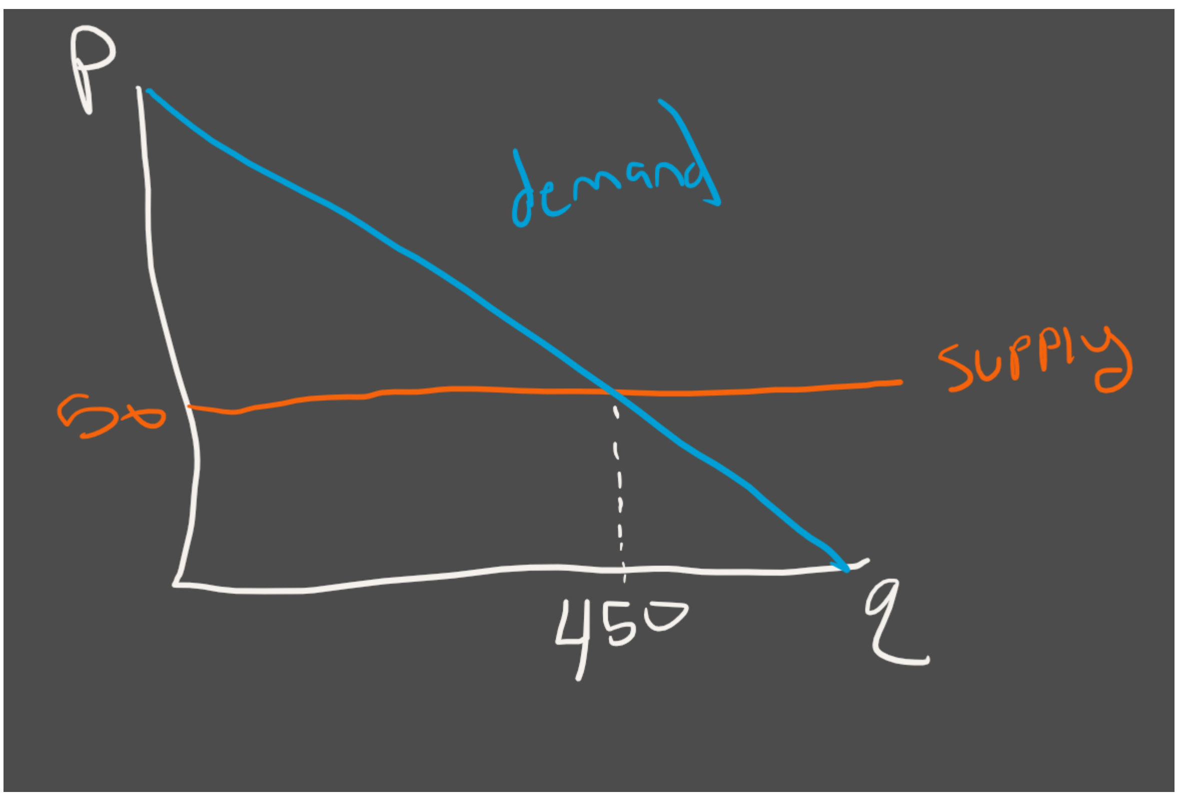
D) Suppose firms are price takers and the price is p. Set up the firm's profit functions.

$$\pi = pq - 50q - 100 = (p - 50)\, q - 100$$

E) Below what price will the firms produce 0?

50

F) Sketch the inverse supply function and the inverse demand function.



G) What is the equilibrium price and quantity in this market?

$$50 = 500 - q$$

$$q = 500 - 50 = 450$$

$$p = 50$$

H) If there is a monopoly that serves this market, what is that firm's profit function?

$$\pi = (500 - q)\, q - (50q + 100)$$

I) What is the monopolist's **optimal quantity** (profit maximizing) to sell?

$$\frac{\partial\left((500 - q)\, q - 50q - 100\right)}{\partial q} = 450 - 2q$$

$$450 = 2q$$

$$q = 225$$

J) What will the monopolist charge for this quantity?

$$p = 500 - (225) = 275$$

K) What is the firm's profit?

$$225 * 275 - (50 * 225 + 100) = 50525$$

L) Why does a monopoly never operate where demand is inelastic?

If demand were inelastic. What happens if they raise price by 1%? Demand decreases by less than 1%. Revenue has to increase.

If demand is inelastic, increasing price a little will ***increase revenue****.*

Costs will go down *because quantity is decreases.*

Thus profit increases.

If a monopolist faces inelastic demand, they will want to increase price and decrease quantity.

M) Show that the firm's price is marked up $\frac{\varepsilon}{1+\varepsilon}$ (where ε is the elasticity of demand) over marginal cost.

Elasticity of demand is how quantity changes in % terms when price increases by 1%.

$$\frac{\frac{\partial q}{q}}{\frac{\partial p}{p}} = \frac{\partial q}{\partial p}\frac{p}{q}$$

$$\frac{\partial\left(500-p\right)}{\partial p}\frac{p}{500-p}$$

$$-1\frac{p}{500-p}$$

Plug in $p = 275$

$$-1\frac{275}{500-275} \quad = -\tfrac{11}{9} = \quad -1.22222$$

If price goes up by 1% demand decreases by 1.2%

Markup is:

$$\frac{-\frac{11}{9}}{1+-\frac{11}{9}} \quad = \quad \frac{11}{2}$$

The firm's marginal cost:

$$\frac{\partial\left(50q+100\right)}{\partial q} \quad = \quad 50$$

$$\frac{11}{2} * 50 = 275$$

This confirms that the monopoly marks up by $\frac{\varepsilon}{1+\varepsilon}$

N) Now let's suppose there are N firms in the market competing in Cournot oligopoly. What are the firm i's profit functions in terms of Q_{-i} (the quantity produced by firms that aren't i) and q_i (firm i's quantity).

$$\pi_i = \left(500 - \left(Q_{-i} + q_i\right)\right) q_i - 50q_i - 100$$

O) What is firm i's optimal quantity in terms of the others firms total Q_{-i}? (Best response function).

$$500q_i - Q_{-i}q_i - q_i^2 - 50q_i - 100$$

$$450q_i - Q_{-i}q_i - q_i^2 - 100$$

$$\frac{\partial\left(\left(500-\left(Q_{-i}+q_i\right)\right)q_i - 50q_i - 100\right)}{\partial q_i} = 450 - Q_{-i} - 2q_i = 0$$

$$450 - Q_{-i} - 2q_i = 0$$

$$q_i = \frac{450 - Q_{-i}}{2}$$

P) Find a symmetric equilibrium in this market (by assuming all firms produce the same amount).

$$q^* = \frac{450 - (N-1)\,q^*}{2}$$

$$q^* = \frac{450}{N+1}$$

Q) What is the market quantity in equilibrium?

$$N * q^* = N\frac{450}{N+1} = \frac{N}{N+1}(450)$$

R) What is the market quantity and price for 2 firms?

$$Q = \frac{2}{3}(450) = 300.$$

$$p = 500 - 300 = 200$$