1 Technology

Consumers. *Preferences. Budget.*

Producers. *Technology.*

How does a firm turn *inputs* into *output*.

Technology of a firm describes how inputs turn into output.

Production Possibilities Sets:

Firm produces cars using, steel, rubber.

$$\{-1, -4, 1\}$$

1.1 Production Functions

Our firms will always use two kinds of inputs and produce one kind of output.

Because of this, we can use **production function** to represent the technology of a firm.

A firm uses 1 steel and 4 rubber to produce 1 car.

$$f(1,4) = 1$$

A firm uses 2 steel and 8 rubber to produce 2 cars.

$$f(2,8) = 2$$

Baker. A baker produces pies using 2 apples and 1 crust per pie. We can represent this technology with:

$$f(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$$

Suppose a new technology is developed that only needs 1 apple per pie,

$$f(x_1, x_2) = min\{x_1, x_2\}$$

Cobb Douglass Production.

$$f(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{1}{4}}$$

1.2 Isoquants

Indifference Cuve. A set of bundles the consumer is indifferent between.

Isoquant. A set of **input bundles** that produce the same amount of output.

1.3 Marginal Products

For consumers, marginal utility (partial derivative of the utility) measures how utility increases as I increase either of the goods while holding the other fixed.

While utility is **ordinal** (the magnitude of the utility number does matter, only comparisons matter), we rarely interpret marginal utility on its own. It is only meaningful in relation to other marginal utilities.

However, production is naturally **cardinal**. The number is meaningful. 1 pie is one pie.

$$min\left\{\frac{1}{2}x_1, x_2\right\}$$

$$2\left(min\left\{\frac{1}{2}x_1, x_2\right\}\right)$$

These productions are **not the same. They represent different technologies.**

Because production functions are cardinal, we can interpret the marginal product.

$$\frac{\partial\left(f\left(x_1, x_2\right)\right)}{\partial x_1} = MP_1$$

This is the marginal product of input 1. It measures how much production increases when we increase the amount of input 1 while hold input 2 fixed.

$$\frac{\partial\left(f\left(x_1, x_2\right)\right)}{\partial x_2} = MP_2$$

This is the marginal product of input 2. It measures how much production increases when we increase the amount of input 2 while hold input 1 fixed.

We can interpret $MP_1 = 1$ roughly as "a one unit increase in input 1 leads to a 1 unit increase in output."

1.4 Diminishing Marginal Product

The property that as add an input, while holding the other input fixed, the extra output I get decreases as we add more and more.

Every unit of a input we add will add less and less to the output.

$$x_1^{\frac{1}{4}}x_2^{\frac{1}{4}}$$

$$MP_1 = \frac{\partial\left(x_1^{\frac{1}{4}}x_2^{\frac{1}{4}}\right)}{\partial x_1} = \frac{1}{4}x_1^{\left(-\frac{3}{4}\right)}x_2^{\frac{1}{4}} = \frac{1}{4}\frac{x_2^{\frac{1}{4}}}{x_1^{\frac{3}{4}}}$$

$$MP_1\left(1,1\right) = \frac{1}{4}\frac{1^{\frac{1}{4}}}{1^{\frac{3}{4}}} = \frac{1}{4}$$

$$MP_1\left(2,1\right) = \frac{1}{4.0}\frac{1^{\frac{1}{4}}}{2^{\frac{3}{4}}} = 0.148651$$

Since the extra productivity I get when I increases good 1 diminishes as I add more good 1, we have **diminishing marginal product.**

$$MP_1 = \frac{1}{4}\frac{x_2^{\frac{1}{4}}}{x_1^{\frac{3}{4}}}$$

$$\frac{\partial\left(\frac{1}{4}\frac{x_2^{\frac{1}{4}}}{x_1^{\frac{3}{4}}}\right)}{\partial x_1} = -\frac{3\sqrt[4]{x_2}}{16x_1^{7/4}} < 0$$

Generic Cobb Douglass

$$x_1^{\alpha}x_2^{\beta}$$

$$\frac{\partial\left(x_1^{\alpha}x_2^{\beta}\right)}{\partial x_1} = \alpha x_1^{\alpha-1}x_2^{\beta}$$

$$MP_1 = \alpha x_1^{\alpha-1}x_2^{\beta}$$

If we take the derivative of this and it negative, there is diminishing marginal product.

$$\frac{\partial\left(\alpha x_1^{\alpha-1}x_2^{\beta}\right)}{\partial x_1} < 0$$

$$\frac{\partial\left(\alpha x_1^{\alpha-1}x_2^{\beta}\right)}{\partial x_1} = (\alpha-1)\alpha x_1^{\alpha-2}x_2^{\beta}$$

$$(\alpha - 1)\alpha x_1^{\alpha-2} x_2^{\beta} < 0$$

$$(\alpha - 1)\alpha < 0$$

$$\alpha < 1$$

as long as $\alpha < 1$ there is diminishing marginal product in x_1.
as long as $\beta < 1$ there is diminishing marginal product in x_2.
Example.

$$f(x_1, x_2) = x_1 + x_2$$

$$MP_1 = 1$$

This has constant marginal product.
If $\frac{\partial(MP_1)}{\partial x_1} < 0$ diminishing marginal product for x_1.
If $\frac{\partial(MP_1)}{\partial x_1} = 0$ constant marginal product for x_1.
If $\frac{\partial(MP_1)}{\partial x_1} > 0$ increasing marginal product for x_1.

1.5 Returns to Scale

Marginal product is about how productivity changes as I change **one of the inputs at a time.**

Returns to scale is about how productivity changes as I change **both** inputs at the same time.

$$x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

This has diminishing marginal product for either input.

$$f(1, 1) = 1^{\frac{1}{2}} 1^{\frac{1}{2}} = 1$$

$$f(2, 2) = 2^{\frac{1}{2}} 2^{\frac{1}{2}} = 2$$

$$f(3, 3) = 3^{\frac{1}{2}} 3^{\frac{1}{2}} = 3$$

....

Constant / Linear Returns to Scale.

$f(2x_1, 2x_2)$ how much output do I get when I double the inputs I'm using.

$2f(x_1, x_2)$ is double my current output.

$$f(2x_1, 2x_2) > 2f(x_1, x_2)$$

"Double inputs and you get more than two times the amount of output." **Increasing Returns to scale.**

Formally for any $t > 1$:

Linear (constant) returns to scale requires: $f(tx_1, tx_2) = tf(x_1, x_2)$.
Decreasing returns to scale requires: $f(tx_1, tx_2) < tf(x_1, x_2)$.
Increasing returns to scale requires: $f(tx_1, tx_2) > tf(x_1, x_2)$.

$$f(x_1, x_2) = x_1 x_2$$

$$f(1, 1) = 1$$

$$f(2, 2) = 4$$

$$f(tx_1, tx_2) = tx_1 tx_2 = t^2 x_1 x_2 = t^2 f(x_1, x_2)$$

$$f(tx_1, tx_2) = t^2 f(x_1, x_2) > tf(x_1, x_2)$$

$$f(tx_1, tx_2) > tf(x_1, x_2)$$

Increasing returns to scale.

$$f(2x_1, 2x_2) = (2x_1)(2x_2) = 4x_1 x_2 = 4f(x_1, x_2)$$

Another example:

$$x_1^{\frac{1}{4}} x_2^{\frac{1}{4}}$$

$$f(2x_1, 2x_2) = (2x_1)^{\frac{1}{4}} (2x_2)^{\frac{1}{4}} = 2^{\frac{1}{4}} 2^{\frac{1}{4}} \left(x_1^{\frac{1}{4}} x_2^{\frac{1}{4}} \right) = 2^{\frac{1}{2}} \left(x_1^{\frac{1}{4}} x_2^{\frac{1}{4}} \right)$$

$$1.4121 \left(x_1^{\frac{1}{4}} x_2^{\frac{1}{4}} \right) < 2 \left(x_1^{\frac{1}{4}} x_2^{\frac{1}{4}} \right)$$

Diminishing marginal product.
For cobb douglass production: $x_1^{\alpha} x_2^{\beta}$
If $\alpha + \beta = 1$ constant returns to scale.
If $\alpha + \beta < 1$ decreasing returns to scale.
If $\alpha + \beta > 1$ increasing returns to scale.

1.6 Technical Rate of Substitution

The TRS is the slope of the isoquants. It measures how much of input 2 (x_2) I can
give up if I am willing to use one more unit of input 1 (x_1).

$$TRS = -\frac{\frac{\partial(f(x_1,x_2))}{\partial x_1}}{\frac{\partial(f(x_1,x_2))}{\partial x_2}}$$

$$TRS = -\frac{\frac{\partial\left(x_1^{\frac{1}{4}} x_2^{\frac{1}{4}} \right)}{\partial x_1}}{\frac{\partial\left(x_1^{\frac{1}{4}} x_2^{\frac{1}{4}} \right)}{\partial x_2}}$$

$$TRS = -\frac{x_2}{x_1}$$

$$TRS = -\frac{\frac{\partial\left(x_1^{\alpha} x_2^{\beta} \right)}{\partial x_1}}{\frac{\partial\left(x_1^{\alpha} x_2^{\beta} \right)}{\partial x_2}} = -\frac{\alpha x_2}{\beta x_1}$$