1 Profit Maximization / Cost Minimization

Production function is a mapping between amounts of input (x_1, x_2) and an amount of output y or q.

$$f(x_1, x_2)$$

$MP_1 = \frac{\partial(f(x_1,x_2))}{x_1}$ how does output increase when I increase x_1 by a little.

$TRS = -\frac{MP_1}{MP_2}$ how much input 2 can I give up if I use one more unit of input 1? Slope of the isoquants (analogous to indifference curves).

1.1 Profit Function

What do firms do?

With consumers "Maximize utility subject to budget constraint."

A firm's goal is to maximize **profit** by choosing x_1 and x_2.

Profit is Revenue - Costs.

Every unit of x_1 costs w_1 and every units of x_2 costs w_2.

w_1 and w_2 are sometimes called "wages".

Revenue is output times **price of output**. Normally there is a link between quantity of output y and price you get for that quantity p. The smaller a firm is, the weaker the link between the quantity they set and the price they get.

Generically, the link between quantity and price is given by $p(f(x_1, x_2))$. This is the price I get for the amount of output I produce by choosing inputs x_1, x_2.

$\pi(x_1, x_2) = f(x_1, x_2)\, p(f(x_1, x_2)) - (x_1 w_1 + x_2 w_2)$

The simplest possible relationship between price and output is that price is fixed at some p regardless of output. **Price taking assumption– associated with perfect competition.** When this is the case, the profit function simplifies to:

$$\pi(x_1, x_2) = f(x_1, x_2)\, p - (x_1 w_1 + x_2 w_2)$$

A firm's goal is maximize this thing by choosing x_1 and x_2.

Suppose $f(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{1}{4}}$, price of output is fixed at p (price-taking assumption) and $w_1 = 1$ and $w_2 = 1$.

$$\pi(x_1, x_2) = p\left(x_1^{\frac{1}{4}} x_2^{\frac{1}{4}}\right) - (x_1 + x_2)$$

To find the maximum two things need to be true:

$$\frac{\partial\left(p\left(x_1^{\frac{1}{4}} x_2^{\frac{1}{4}}\right) - (x_1 + x_2)\right)}{\partial x_1} = 0$$

$$\frac{p\sqrt[4]{x_2}}{4x_1^{3/4}} - 1 = 0$$

$$\frac{\partial\left(p\left(x_1^{\frac{1}{4}}x_2^{\frac{1}{4}}\right) - (x_1 + x_2)\right)}{\partial x_2} = 0$$

$$\frac{p\sqrt[4]{x_1}}{4x_2^{3/4}} - 1 = 0$$

We need to solve these at the same time:

$$\frac{p\sqrt[4]{x_2}}{4x_1^{3/4}} - 1 = 0, \frac{p\sqrt[4]{x_1}}{4x_2^{3/4}} - 1 = 0$$

This solution gives us the Profit maximizing input levels:

$$x_1 = \frac{p^2}{16}, x_2 = \frac{p^2}{16}$$

To profit maximizing level of output:

$$y^* = \left(\frac{p^2}{16}\right)^{\frac{1}{4}}\left(\frac{p^2}{16}\right)^{\frac{1}{4}}$$

$$y^* = \frac{p}{4}$$

1.2 Short Run Profit Maximization

A market is in the **long run**, if both inputs can be adjusts.

A market is in the **short run**, if some input is fixed.

Example. Suppose

Suppose $f(x_1, x_2) = x_1^{\frac{1}{4}}x_2^{\frac{1}{4}}$, price of output is fixed at p (price-taking assumption) and $w_1 = 1$ and $w_2 = 1$. But x_2 is fixed at 16.

$$\pi(x_1, x_2) = p\left(x_1^{\frac{1}{4}}x_2^{\frac{1}{4}}\right) - (x_1 + x_2)$$

Since $x_2 = 16$ we plug this into the profit function. We get the short run profit:

$$\pi(x_1, 16) = p\left(x_1^{\frac{1}{4}}(16)^{\frac{1}{4}}\right) - (x_1 + 16)$$

$$\pi\left(x_1, 16\right) = 2px_1^{\frac{1}{4}} - x_1 - 16$$

$$\frac{\partial\left(2px_1^{\frac{1}{4}} - x_1 - 16\right)}{\partial x_1} = 0?$$

$$2p\frac{1}{4}x_1^{-\frac{3}{4}} - 1 = 0$$

$$p\frac{1}{2}x_1^{-\frac{3}{4}} = 1$$

$$x_1^{-\frac{3}{4}} = \frac{2}{p}$$

$$x_1^{\frac{3}{4}} = \frac{p}{2}$$

$$x_1 = \left(\frac{p}{2}\right)^{\frac{4}{3}}$$

1.3 Profit Maximization Requires Cost Minimization

$\pi\left(x_1, x_2\right) = f\left(x_1, x_2\right)p\left(f\left(x_1, x_2\right)\right) - \left(x_1w_1 + x_2w_2\right)$

If I'm maximizing profit, I always produce **some** level of output y. If I wasn't cost minimizing, there is some cheaper way to produce y. If I choose the cheaper way of producing, it does change revenue, but it decreases cost and thus increases profit. Thus, **profit maximization requires that I am producing output in the cheapest way possible.**

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