1 Cost Min / Profit Max

1.1 Profit Max

$$\pi(x_1, x_2) = f(x_1, x_2)\, p(f(x_1, x_2)) - (x_1 w_1 + x_2 w_2)$$

If the price of output p does not depend on the quantity produced. **Price taking assumption.** The more competition there is in a market (the more firms produce the same thing) the more reasonable this assumption is.

$$\pi(x_1, x_2) = f(x_1, x_2)\, p - (x_1 w_1 + x_2 w_2)$$

In the long run, we can choose both x_1 and x_2. To find the profit maximizing inputs,

$$\frac{\partial(\pi(x_1, x_2))}{\partial x_1} = 0$$

$$\frac{\partial(\pi(x_1, x_2))}{\partial x_2} = 0$$

These are the necessary conditions for profit max. In our class they will also be sufficient for maximum.

In the **short run**, some input is fixed. Let's say it is x_2. We can only adjust x_1. In this case we only have to worry about this condition being true:

$$\frac{\partial(\pi(x_1, x_2))}{\partial x_1} = 0$$

The only time I invite you to maximize profit this way (by choosing x_1 and x_2 to maximize $\pi(x_1, x_2)$ is in the short run where some input is fixed.

1.2 Profit Maximization Requires Cost Minimization

$\pi(x_1, x_2) = f(x_1, x_2)\, p(f(x_1, x_2)) - (x_1 w_1 + x_2 w_2)$

If I'm maximizing profit, I always produce **some** level of output y. If I wasn't cost minimizing, there is some cheaper way to produce y. If I choose the cheaper way of producing, it does change revenue, but it decreases cost and thus increases profit. Thus, **profit maximization requires that I am producing output in the cheapest way possible.**

We can break down profit maximization into two simpler steps.

1. **Find the cheapest way to produce any level of output y. (Cost Min).**

2. Find the optimal level of output.

1.3 Cost Minimization

The firm wants to solve this problem:

Minimize $(x_1 w_1 + x_2 w_2)$ subject to $f(x_1, x_2) = y$.

With utility maximization found the highest indifference curve that was on the budget line.

Isoquants

A set of input bundles that produce the same output.

$$f(x_1, x_2) = y$$

Isocosts

Set of input bundles that cost the same amount.

$$w_1 x_1 + w_2 x_2 = c$$

To miminize cost, we look for a bundle where the **isocost curve through a bundle just touches but does not cross through the isoquant.**

1.4 Minimizing Cost for a Cobb-Douglas Production Function

$f(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{1}{4}}$ and $w_1 = 1$ and $w_2 = 1$.

Want to find a point where the slope of the isoquant is the same as the slope of the isocost.

$$w_1 x_1 + w_2 x_2 = c$$

This looks a lot like:

$$p_1 x_1 + p_2 x_2 = m$$

Want to find a point where the slope of the isoquant is the same as the slope of the isocost.

$$TRS = -\frac{w_1}{w_2}$$

$$-\frac{\frac{\partial\left(x_1^{\frac{1}{4}} x_2^{\frac{1}{4}}\right)}{\partial x_1}}{\frac{\partial\left(x_1^{\frac{1}{4}} x_2^{\frac{1}{4}}\right)}{\partial x_2}} = -1$$

$$-\frac{\frac{1}{4}x_1^{-\frac{3}{4}}x_2^{\frac{1}{4}}}{\frac{1}{4}x_1^{\frac{1}{4}}x_2^{-\frac{3}{4}}} = -1$$

$$-\frac{x_2}{x_1} = -1$$

$$x_2 = x_1$$

Constraint #1 equal slope constraint $x_2 = x_1$
Constraint #2 production constraint $x_1^{\frac{1}{4}}x_2^{\frac{1}{4}} = y$
Plugging #1 into #2:

$$x_1^{\frac{1}{4}}x_1^{\frac{1}{4}} = y$$

$$x_1^{\frac{1}{2}} = y$$

$$x_1 = y^2$$

$$x_2 = y^2$$

These are called the **conditional factor demands**.

$$\left(y^2, y^2\right)$$

This is the cheapest possible way to produce output y.
Check how much output this produces:

$$f\left(y^2, y^2\right) = \left(y^2\right)^{\frac{1}{4}}\left(y^2\right)^{\frac{1}{4}} = \left(y^{\frac{1}{2}}\right)\left(y^{\frac{1}{2}}\right) = y$$

1.5 The Cost Function

The cost function is the cost of producing output y in the cheapest possible.
Let x_1^* and x_2^* be the conditional factor demands for producing y.

$$c(y) = w_1(x_1^*) + w_2(x_2^*)$$

In the example above since $x_1^* = y$ and $x_2^* = y$ and $w_1 = w_2 = 1$

$$c(y) = y^2 + y^2 = 2y^2$$

1.6 Profit Maximization Through Cost Minimization

$$\pi(y) = p(y)\,y - c(y)$$

Suppose we make the price-taking assumption:

$$\pi(y) = py - c(y)$$

For our example above:

$$\pi(y) = py - 2y^2$$

We can now maximize by choosing only y by finding where the derivative of this is zero.

$$\frac{\partial\left(py - 2y^2\right)}{\partial y} = 0$$

$$p - 4y = 0$$

$$4y = p$$

$$y^* = \frac{1}{4}p$$

This is the profit maximizing level of output.

If you need to find the amount of inputs used at the profit-maximizing level, plug the optimal output back into the conditional factor demands.

The bundle of inputs we want to use to maximize profit is:

$$\left(\left(\frac{1}{4}p\right)^2, \left(\frac{1}{4}p\right)^2\right)$$

$$\left(\frac{1}{16}p^2, \frac{1}{16}p^2\right)$$

1.7 Example with Perfect Complements Production

A firm produces pies that have price p by using 2 apples and 1 curst for every pie. $w_1 = 1$ $w_2 = 1$.

$$f(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$$

Our first condition is the "no waste condition".

$$\frac{1}{2}x_1 = x_2$$

Our second condition is the production condition:

$$min\left\{\frac{1}{2}x_1, x_2\right\} = y$$

Plug condition 1 into condition 2:

$$min\{x_2, x_2\} = y$$

$$x_2 = y$$

$$x_1 = 2y$$

These are out conditional factor demands.
Cost function ($w_1 = 1$,$w_2 = 1$)

$$w_1 2y + w_2 y$$

$$c(y) = 2y + y = 3y$$

1.8 What can go wrong– Linear/Increasing Returns to Scale

Let's set up the profit function:

$$\pi(y) = py - 3y = y(p - 3)$$

Suppose $p = 2$

$$\pi(y) = -y$$

Suppose $p = 4$

$$\pi(y) = y$$

This is more generally a problem with increasing returns to scale.

If we have increasing returns to scale, if there is any level of output that produces positive profit, then there is no profit maximizing level output. The firm wants to produce an infinite amount.

An exercise to try:

Suppose $f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$. Price of output is $p = 100$ and $w_1 = 1$, $w_2 = 1$

1.9 More on Supply Under Price-Taking

$$\pi(y) = py - c(y)$$

To find the profit maximizing level, we look for a place where the marginal profit is equal to zero.

$$\frac{\partial(\pi(y))}{\partial y} = 0$$

$$\pi = rev - cost$$

$$\frac{\partial rev}{\partial y} - \frac{\partial cost}{\partial y} = 0$$

$$\frac{\partial rev}{\partial y} = \frac{\partial cost}{\partial y}$$

$$mr = mc$$

Marginal revenue is equal to marginal cost has to be true for every firm at the optimum regardless of what we assume about how price changes when we change output.

Under price taking profit:

$$py - c(y)$$

Marginal revenue:

$$p$$

Marginal cost:

$$\frac{\partial c\left(y\right)}{\partial y}$$

Specifically for price taking firms:

$$p = mc\left(y\right)$$

Price equals marginal cost is identical to marginal revenue is equal marginal cost **only when we make the price taking assumption**.