0.1 Sustaining Cooperation In Repeated Interactions.

How can we possibly sustain non-equilibrium outcomes in game theory? How can we sustain cooperation in the prisoner's dilemma.

Let's suppose the players in a prisoner's dilemma play the game every day.

	cooperate	deviate
cooperate	4,4	0,10
deviate	10,0	**2,2**

Figure 1: A simplified prisoner's dilemma game.

Suppose we play this game every day. I care about tomorrow β as much as I care about today.

Suppose we just play $(deviate, deviate)$ every day.

$$2 + \beta 2 + \beta^2 2 + \beta^3 2 + ...$$

$$= 2 \sum_{t=0}^{t} \beta^t$$

A Fact. as long as $0 < \beta < 1$:

$$\sum_{t=0}^{t} \beta^t = \frac{1}{1-\beta}$$

Utility of both deviating forever:

$$= 2 \sum_{t=0}^{t} \beta^t = \frac{2}{1-\beta}$$

If $\beta = 0.5$

$$\frac{2}{1-0.5} = 4$$

If $\beta = 0.99$

$$\frac{2}{1-0.99} = 200.$$

Because deviate deviate is a nash equilibrium, it is the default outcome in absence some agreement.

What kind of agreement is self-enforcing and can sustain cooperation?

Grim Trigger Strategy (Agreement).

"We cooperate unless one of us has ever deviated in the past. If someone has deviated, we both play deviate forever. "

On any day we are supposed to cooperate. I ask "should I go along with the agreement and cooperate or should I deviate?"

What happens if I cooperate?

$$4 + \beta 4 + \beta^2 4 + \beta^3 4 + ... = 4 \sum_{t=0}^{\infty} \beta^t$$

$$= 4 \frac{1}{1-\beta}$$

What happens if I deviate?

$$10 + \beta 2 + \beta^2 2 + \beta^3 2 + ... = 10 + \beta \sum_{t=0}^{\beta} 2$$

$$10 + \left(\beta 2 + \beta^2 2 + \beta^3 2 + ...\right) = 10 + \beta \left(2 + \beta 2 + \beta^2 2 + ...\right)$$

$$= 10 + 2\beta \sum_{t=0}^{\beta} \beta^t = 10 + \frac{2\beta}{1-\beta}$$

Is it better to cooperate or deviate. It is better to cooperate:

$$4 \frac{1}{1-\beta} > 10 + \frac{2\beta}{1-\beta}$$

$$4 > 10 (1-\beta) + 2\beta$$

$$4 > 10 - 10\beta + 2\beta$$

$$\beta > \frac{3}{4}$$

As long as the players are "patient enough" they sustain cooperation.

1 Externalities

When the actions of one person affect the outcomes or utility of other people, we say there is an externality.

Pollution- Negative Externality.

Mowing The Grass- Positive Externality

1.1 Tragedy of the Commons Example

Negative Externality.

There is a lake and some fishing boats. Limited number of fish in the lake.

$100\sqrt{B}$ fish will be caught on a lake when B boats are fishing.

Fish can be sold for \$1 but it costs \$10 to buy fuel and supplies to fish.

The number of fish each boat catches is:

$$\left(\frac{100\sqrt{B}}{B}\right)$$

Profit of each boat on the lake:

$$1\left(\frac{100\sqrt{B}}{B}\right) - 10$$

1.1.1 Equilibrium Number of Boats

How many boats will be on the lake?

If profit is positive, there is no incentive for boats to leave the lake.

$$1\left(\frac{100\sqrt{B}}{B}\right) - 10 \geq 0$$

$$0 < B \leq 100$$

We expect 100 boats to be on the lake. If there are fewer than 100, there is an incetive for more to enter because they can earn positive profit. If there are 100 or more, there is no incentive to enter. If there are more than 100, there is incentive to leave.

$B = 100$

Total fish: 1000

Fish per boat: 10

Profit per boat: 0

Total market profit: 0

1.1.2 Socially Optimal Number of Boats

Maximize the total profit earned by boats on the lake. Total profit function:

$$100\sqrt{B} - 10B$$

Maximize total profit:

$$\frac{\partial \left(100\sqrt{B} - 10B\right)}{\partial B} = \frac{50}{\sqrt{B}} - 10$$

$$\frac{50}{\sqrt{B}} = 10$$

$$B = 25$$

Total profit earned:

$$100\sqrt{25} - 10 * 25 = 250$$

1.1.3 Fishing Fee

How can the government ensure there is not overfishing?

We know the socially optimal number of boats it 25. What if the government sets a license fee to fish of t? Now the profit of a boat needs to exceed the fishing fee for a boat to want to enter. The government finds a fee t such that when the socially optimal number enter (25) they earn zero profit.

$$1\left(\frac{100\sqrt{25}}{25}\right) - 10 = t$$

$$t \to 10$$

If the government charges 10, we get the socially optimal number of boats. Each boat earns zero as before, but now at least the government has 250 in revenue to use for things like improving the lake.