1 Monopoly

Perfect competition (price-taking assumption). Each firm is **so small** that changes in their quantity have no impact on the market price (price they can get for what they produce). They take price as fixed and constant at c.

Monopoly is when there is just one firm that produces that thing. Because this is the only firm that produces "that thing", we can't assume that it takes prices as fixed.

1.1 Monopolies and the Price-Taking Assumption

Suppose demand is: $q(p) = \frac{100}{p}$.
What is the most a monopolist could charge for selling q? They can charge whatever consumers are willing to pay to buy q units. This is literally the definition of **inverse demand**.

$$q = \frac{100}{p}$$

$$p = \frac{100}{q}$$

This is the inverse demand. For instance, if they want to sell 50 units, the most they can charge is 2. If they want to sell 100 units the most they can charge is 1.

1.2 The Monopolist's Profit Function

This is the price-taking profit:

$$\pi(q) = pq - c(q)$$

For the monopolist (let $p(q)$ be the inverse demand which is the most they can charge for q)

$$\pi(q) = p(q)\, q - c(q)$$

For all firms, regardless of the competition structure they are under. You still have to find where **marginal profit is zero**.

$$\frac{\partial(\pi(q))}{\partial q} = 0$$

Since profit is revenue minus cost:

$$\frac{\partial Rev}{\partial q} - \frac{\partial Cost}{\partial q} = 0$$

$$\frac{\partial Rev}{\partial q} = \frac{\partial Cost}{\partial q}$$

We get “marginal revenue is equal to marginal cost”:

$$MR = MC$$

For a firm in **perfect competition**, revenue is pq and so marginal revenue is p. You get \$$p$ for every unit you sell. So the profit maximizing condition is:

$$p = mc$$

For a monopolist, revenue $p(q)q$. $\frac{\partial(p(q)q)}{\partial q} = p'(q)q + p(q)$
Marginal revenue has two parts:

$$p(q) + p'(q)q$$

This has two parts, the direct effect of increasing quantity is that you sell more at some price $(p(q))$ indirect effect that as you increase quantity, the price goes down $(p'(q)q)$.
The monologist takes advantage of the fact they can “control” the price, by lowering quantity to drive up the price.

1.3 Example of Maximizing Profit

Suppose demand is $q_d(p) = 100 - p$. Cost is $c(q) = 10q$.
Let start with a perfect competition baseline. The firm takes price as fixed at p:

$$\pi(q) = pq - 10q$$

What does a firm want to do here?

$$\pi(q) = (p - 10)q$$

Suppose $p > 10$. For instance, suppose it is $p = 11$:

$$\pi(q) = q$$

Since they will sell as much as you want at any price above 10, the inverse supply is a horizontal line at $p = 10$.

Let's find the equilibrium. The inverse demand:

$$q = 100 - p$$

$$p = 100 - q$$

Since the supply curve is a horizontal line at $p = 10$:

$$10 = 100 - q$$

$$q = 90$$

In perfect competition, the quantity is 90 and the price is 10.

Another way to solve this is that in perfect competition, we know that price is equal to marginal cost. Which here is 10.

$$p = 10$$

Consumers demand 90 at a price of 10.

In perfect competition $p = 10, q = 90$.

Notice that the firm's profit is zero. Because each unit costs $10 and they get $10 for each unit as well.

Monopolist:

$q_d(p) = 100 - p$. Cost is $c(q) = 10q$. The inverse demand is $p = 100 - q$.

$$\pi(q) = (100 - q) q - 10q$$

$$= 100q - q^2 - 10q$$

To maximize this, find where the derivative is zero:

$$100q - q^2 - 10q$$

$$\frac{\partial(\pi(q))}{\partial q} = 100 - 2q - 10$$

Set this to zero:

$$90 - 2q = 0$$

$$q = 45$$

How much can they charge? Plug this quantity back into the inverse demand:

$$p = (100 - q)$$

$$p = 100 - 45 = 55$$

The firm's profit:

$$(45)(55) - 10(45) = 2025$$

1.4 What does a monopoly do?

Monopolists love inelastic demand.

With inelastic demand a 1% increase in price leads to a **less** than 1% decrease in demand.

Suppose the firm increases price by 1%. What happens to revenue?

Whenever demand is inelastic, a 1% increase in price has to increase revenue.

Since the increase in price decreases quantity, cost also go down.

Revenue goes up and **costs go down**, **profit has to go up**.

A monopolist can never be operating in the inelastic portion of the demand curve. Choosing a price where demand is inelastic cannot be optimal.

1.5 Maximizing profit with a constant unit elastic demand.

Suppose demand $\frac{\frac{1}{2}m}{p}$ and cost is $c(q) = q$

$$\frac{\partial\left(\frac{\frac{1}{2}m}{p}\right)}{\partial p}\frac{p}{\frac{\frac{1}{2}m}{p}} = -1$$

$$q = \frac{\frac{1}{2}m}{p}$$

$$p = \frac{\frac{1}{2}m}{q}$$

$$\pi\left(q\right) = \frac{\frac{1}{2}m}{q}q - q$$

$$\frac{1}{2}m - q$$

This is "maximized" when $q = 0$ and price is infinite.

1.6 Markup

Recall that in perfect competition, $p = mc$.

Suppose ϵ is the elasticity of demand at the optimal quantity then the following markup equation holds:

$p = \frac{\epsilon}{1+\epsilon}mc$.

1.7 In Action

Suppose the firm has a constant mc of 1. $c\left(q\right) = q$

Suppose demand is $q = \frac{100}{p^2}$.

$$\frac{\partial\left(\frac{100}{p^2}\right)}{\partial p}\frac{p}{\frac{100}{p^2}} = -2$$

We can use the markup equation to find the optimal price:

$$p = \frac{\epsilon}{1+\epsilon}mc$$

The optimal price:

$$p = \frac{-2}{1+(-2)}1 = 2$$

1.8 Markup in Empirical Work

This equation allows us to calculate any of the three values ϵ, p, mc while only observing two of them. For instance, suppose we observe a monopolist changes 10 and that consumer demand elasticity is -1.5. We can determine the underlying marginal cost:

$$10 = \frac{-1.5}{1 + (-1.5)} mc$$

$$3.33333 = mc$$

1.9 Note on operating in the elastic portion of the demand curve.

In the example above with $q = 100 - p$ and $c(q) = 10p$, we found the firm operated where $p = 55$. Let's check this is in the inelastic portion of the demand curve:

The elasticity is:

$$\frac{\partial (100 - p)}{\partial p} \frac{p}{100 - p} \quad = \quad -\frac{p}{100 - p}$$

This is inelastic when $-\frac{p}{100-p} < -1$ which occurs where:

$$50 < p < 100$$