0.1 Consumer Surplus Under Monopoly

A monopolist has cost function $c(q) = 10q$ and demand is $q = 100 - p$.

Profit function. We need the inverse demand (what is the most that consumers will pay to buy q units?)

$$q = 100 - p$$

$$p = 100 - q$$

$$\pi(q) = (100 - q)\, q - 10q$$

At the maximum, marginal profit has to be zero:

$$\frac{\partial\left((100 - q)\, q - 10q\right)}{\partial q} = 0$$

$$90 - 2q = 0$$

$$q = 45$$

$$p = 100 - 45 = 55$$

The monopolist maximizes profit by selling 55 units at a price of 45.

1 Monopoly Behavior

1.1 Types of Price Discrimination

- First Degree Price Discrimination.
  - Charge everyone exactly what they are willing to pay for the good.
  - Requires two things: know what everyone is willing pay (identify individuals), be able to charge them different amounts.
  - All of the possible surplus is captured by the monopolist. There is no dead-weight loss. The outcome is efficient, and all the of the possible surplus is captured by the monopolist.
  - This is the most extreme form of price discrimination. This is more of a thought experiment about what is possible rather than something that really happens.
- Second Degree Price Discrimination.

  - We cannot identify consumers or types of consumers, but you can offer different packages of quality, quantity to get different amounts of money from different types of people.
  - The firm makes the different options available but does force anyone into buying a particular option.
  - Quality: Qualities whiskey, regular/reserve wines, first-class/coach travel.
  - Levels of service.
  - Wifi tiers.

- Third Degree Price Discrimination.
  - Types of consumes that can be identified and have different kinds of demand
  - Different types *have* to pay different amounts.
  - Student discounts. Senior discounts.
  - Leverage different elasticies of demand and charge different amounts to people with more/less elastic demand. People who have more elastic demand get changed less.

- Bundling.
  - A firm that sells several different "goods".
  - The firm forces consumers to buy these goods in a bundle.
  - Microsoft office. You can't buy powerpoint or word separately.
  - Cable service.

- Two-Part Tariff.
  - In a situation where an individual might demand more than unit of a thing:
  - Create a low unit cost, and then charge an up-front fee for the right to buy at that lower price.
  - Theme park tickes. Don't charge you per ride, just charge entry fee.
  - "Free" coffee mug. $15 a month for free coffee.
  - Amazon price.

1.2 First Degree Price Discrimination in Action

Suppose there are three people willing to pay $3, $2, $1 for a good respectively. Let's assume the firm has no cost. $c(q) = 0$.

No price discrimination (everyone pays the same price).

Price	# Buyers	Profit
\$3	1	\$3
\$2	**2**	**\$4**
\$1	3	\$3

Under selling at a **single price** the most I can earn is \$4. Under first-degree price discrimination sell to everyone at their willinness to pay:

$$\pi = \$3 + \$2 + \$1 = \$6$$

1.3 Third Degree Price Discrimination in Action

Suppose there are two groups of people: students and non-students. A movie theater sells tickets to both groups. Assume the firm has zero marginal cost so that $c(y) = 0$.

Students have demand function: $y_s = 100 - 2p$

Non-students have demand function: $y_n = 100 - p$.

Students have more elastic demand than non-students.

For fomality, let's calculate the elasticities:

Elasticity for students:

$$\frac{\partial (100 - 2p)}{\partial p} \frac{p}{100 - 2p} = -\frac{2p}{100 - 2p}$$

Elasticity for non-students:

$$\frac{\partial (100 - p)}{\partial p} \frac{p}{100 - p} = -\frac{p}{100 - p}$$

To show that students have more elastic demand:

$$-\frac{2p}{100 - 2p} < -\frac{p}{100 - p}$$

$$0 < p < 50$$

Since these are really the relevant set of prices for students, their demand will always be more elastic. Because of this, we can predict that the theater will charge students less.

Calculate profit under no price discrimination:

Total demand: $y_s + y_n = 100 - 2p + 100 - p = 200 - 3p$

$$y = 200 - 3p$$

Inverse total demand:

$$p = \frac{200 - y}{3}$$

Profit function for everyone:

$$\pi(y) = y\left(\frac{200-y}{3}\right)$$

Let's maximize this:

$$\frac{\partial\left(y\left(\frac{200-y}{3}\right)\right)}{\partial y} = 0$$

$$\frac{\partial\left(\frac{200}{3}y - \frac{1}{3}y^2\right)}{\partial y} = 0$$

$$\frac{200}{3} - \frac{2y}{3} = 0$$

$$\frac{2}{3}y = \frac{200}{3}$$

$$y = 100$$

To get the price, we plug this back into the inverse demand:

$$p = \frac{200-y}{3}$$

$$p = \frac{200-100}{3} = 33.3333$$

$$\pi = 100\,(33.3333) = 3333.33$$

1.3.1 Third Degree Price Discrimination Student Price

Students have demand function: $y_s = 100 - 2p$

Inverse demand: $p = 50 - \frac{1}{2}y_s$

Profit for students:

$$\pi(y_s) = y_s\left(50 - \frac{1}{2}y_s\right)$$

$$= 50y_s - \frac{1}{2}y_s^2$$

To maximize this profit, look for where it has zero marginal profit:

$$\frac{\partial\left(50y_s - \frac{1}{2}y_s^2\right)}{\partial y_s} = 0$$

$$50 - y_s = 0$$

$$y_s = 50$$

It is optimal to sell 50 student tickets. We can charge:

$$p = 50 - \frac{1}{2}(50)$$

$$p = 25$$

Profit from students:

$$50 * 25 = 1250$$

1.3.2 Third Degree Price Discrimination Non-Student Price

Non-students have demand function: $y_n = 100 - p$.

Inverse demand: $p = 100 - y_n$

$$\pi(y_n) = y_n(100 - y_n)$$

$$\frac{\partial(y_n(100 - y_n))}{\partial y_n} = 0$$

$$100 - 2y_n = 0$$

$$y_n = 50$$

It is also optimal to sell 50 non-student tickets. We can charge them:

$$p_n = 100 - 50 = 50$$

Profit for non-students:

$$50 * 50 = 2500$$

1.3.3 Comparing Profit

Total profit from price discrimination:

$$1250 + 2500 \quad = \quad 3750$$

Total profit without price discrimination:

$$= 3333.33$$

1.4 Bundling

Suppose we have a clothing company that sells shirts and pants. Two consumers that have the following willingness to pay for the types of clothing. Let's ignore costs by setting cost to zero.

	Shirt	Pants	Bundle
Consumer 1	50	30	80
Consumer 2	10	80	90

Let's suppose we sell separately.
What the most I can earn by selling shirts?
If you want to sell two shirts, what is highest price you can charge? 10

$$10 * 2 = 20$$

If you want to sell one shirts, what is highest price you can charge? 50

$$50 * 1 = 50$$

What the most I can earn by selling pants?
If you want to sell two pairs of pants, what is highest price you can charge?

$$30 * 2 = 60$$

If you want to sell one pair pants, what is highest price you can charge? 80

$$80 * 1 = 80$$

The most I can earn is 80
Suppose I force them to buy an outfit (bundle).
Sell two bundles? What is the most I can charge? 80

$$2 * 80 = 160$$

Compare to the profit under selling separately:

$$160 > 80 + 50 = 130$$

1.5 Two-Part Tariff

For example, suppose a consumer's demand for coffee is $q = 10 - p$ and the firm has zero marginal cost for coffee.