1 Monopoly Behavior

1.1 Two-Part Tariff

The two part tariff is possible when consumers demand more than one unit of a good.

Membership fees, Amazon Prime, Season Tickets,

The firms sets a very low price ($p = mc$) in order to maximize consumer surplus. Then the firm charges an up-front fee equal to that consumer surplus.

This allows the monopolist to capture the entire possible surplus as profit just like in first-degree price discrimination.

For example, suppose a consumer's demand for coffee is $q = 10 - p$ and the firm has zero marginal cost for coffee.

In the firm has to charge one price for every cup, the best it can do is charge \$5 and sell 5 cups for profit of \$25.

If it sets price of coffee to zero. The consumer surplus is 50. The firm then charges \$50 up front to capture this entire surplus.

2 The Cournot Model of Competition

Perfect competition, there are *many* firms.

Monopoly, there is *one* firm.

Oligopoly, there are *some* firms.

2.1 Extending the Monopoly Model

The monopolist profit:

q is their quantity

$p(q)$ is the inverse demand which tells the most they could charge for selling q.

$$\pi(q) = qp(q) - c(q)$$

A monopolists profit maximizing decision doesn't depend on what any other firm does.

Suppose we have two firms:

q_1 is firm $1s$ quantity

q_2 is firm $2s$ quantity

$$\pi_1(q_1, q_2) = q_1 p(q_1 + q_2) - c(q_1)$$

$$\pi_2(q_1, q_2) = q_2 p(q_1 + q_2) - c(q_2)$$

Firm 1 wants to find where it's marginal profit is zero.

$$\frac{\partial\left(q_1 p\left(q_1+q_2\right)-c\left(q_1\right)\right)}{\partial q_1}=0$$

$$p\left(q_1+q_2\right)+q_1 \frac{\partial p\left(q_1+q_2\right)}{\partial q_1}=\frac{\partial c\left(q_1\right)}{\partial q_1}$$

Notice that this first order condition depends on q_2. So the optimal choice for firm one (q_1) depends on what q_2.

Similarly, firm 2's optimal action depends on firm 1s choice.

2.2 Game Theory

Game theory is the study of strategy.

This model is a **Game**.

Games consist of three parts:

1) Players

Firms

2) Actions available to the players.

They can choose a quantity q_i

3) Payoffs which determine how much a player gets based the entire set of actions chosen by all players.

Profit function.

2.3 Example of Maximizing Profit with Two Firms

Market quantity $Q=q_1+q_2$. Inverse demand $p(Q)=100-Q$. Cost function for each firm is $c\left(q_i\right)=10 q_i$.

Since $Q=q_1+q_2$ we can write the inverse demand:

$$p\left(q_1+q_2\right)=100-\left(q_1+q_2\right)$$

Now we can write both firm's profit functions:

$$\pi_1\left(q_1, q_2\right)=q_1\left(100-\left(q_1+q_2\right)\right)-10 q_1$$

$$\pi_2\left(q_1, q_2\right)=q_2\left(100-\left(q_1+q_2\right)\right)-10 q_2$$

What is the optimal choice for firm 1?

It solves the following first-order condition:

$$\frac{\partial \left(q_1 \left(100 - (q_1 + q_2)\right) - 10q_1\right)}{\partial q_1} = 0$$

$$-2q_1 - q_2 + 90 = 0$$

$$q_1 = \frac{90 - q_2}{2}$$

This is a **best response function.** It tells us for any action taken by firm 2, what is firm 1's best action (best response).

Suppose firm 1 believes firm 2 will choose $q_2 = 50$

$$q_1 = \frac{90 - 50}{2} = 20$$

Since the two firms have same sort of profit function, the best response for firm 2:

$$q_2 = \frac{90 - q_1}{2}$$

What is firm 2's best response to 20?

$$q_2 = \frac{90 - 20}{2} = 35$$

If firm 2 plays 50, then firm 1 best responds with 20, then firm 2's best response to 20 is not 50, it's 35. There is some inconsistency.

To find a consistent set of actions, we really need to find quantities that are **mutually best responses.**

In this case, (q_1, q_2).

We need to find a pair that solve the best response functions at the same time:

$$Solve[\{q_2 == \frac{90 - q_1}{2}, q_1 == \frac{90 - q_2}{2}\}, \{q_1, q_2\}]$$

$$q_1 = 30, q_2 = 30$$

Let's check these are mutually best responses.

Firm 2's best response to 30

$$q_2 = \frac{90 - 30}{2} = \frac{60}{2} = 30$$

Firm 1's best response to 30

$$q_1 = \frac{90 - 30}{2} = \frac{60}{2} = 30$$

In this case, we have a pair of quantities where neither firm has incentive to change what they are doing.

When we find actions that mutually best responses we are really solving for a type of equilibrium.

Nash Equilibrium. Is a set of actions that are mutually best responses to eachother.

Here, the *only* Nash equilibrium is $q_1 = 30, q_2 = 30$.

This is a **symmetric** nash equilibrium because all players choose the same action.

For our cournot problems, every game will have a symmetric equilibrium if the firms have the same cost function.

2.4 Equilibrium with N firms.

Now suppose we have N firms.

Firm i's quantity: q_i

Market (total) quantity: $Q = \sum_{i=1}^{N} q_i$

Total quantity i is not responsible for: $Q_{-i} = Q - q_i$

Notice that $Q = q_i + Q_{-i}$

We can write each firm's profit:

$$\pi_i\left(q_i, Q_{-i}\right) = q_i p\left(q_i + Q_{-i}\right) - c\left(q_i\right)$$

Each firm's best response is where this profit has zero slope:

$$\frac{\partial\left(q_i p\left(q_i + Q_{-i}\right) - c\left(q_i\right)\right)}{\partial q_i} = 0$$

This will give us a best response for each firm.

2.5 From Example Above

N firms. Inverse demand $p\left(Q\right) = 100 - Q$. Cost function for each firm is $c\left(q_i\right) = 10q_i$.

$$\pi_i\left(q_i, Q_{-i}\right) = q_i\left(100 - \left(q_i + Q_{-i}\right)\right) - 10q_i$$

Firm i's optimal quantity occurs where the slope of this is zero with respect to q_i

$$\frac{\partial \left(q_i \left(100-\left(q_i+Q_{-i}\right)\right)-10q_i\right)}{\partial q_i}=0$$

Let's simplify the profit:

$$100q_i - q_i \left(q_i + Q_{-i}\right) - 10q_i$$

$$90q_i - q_i^2 - q_i Q_{-i}$$

The derivative and set equal to zero:

$$90 - 2q_i - Q_{-i} = 0$$

Solve for q_i to get the best response:

$$q_i = \frac{90 - Q_{-i}}{2}$$

We know all the firms have this best response function.

What if we look for a $q = q_1 = q_2 = ... = q_n$ that solves all N of the best response functions at the same time?

After finding the best response function, we impose symmetry to get this:

All firms set $q_i = q$

$Q_{-i} = (N-1)\,q$

$$q = \frac{90 - (N-1)\,(q)}{2}$$

Solve this for q to get the equilibrium:

$$2q + (N-1)\,(q) = 90$$

$$(2 + N - 1)\,q = 90$$

$$(N+1)\,q = 90$$

$$q = \frac{90}{N+1}$$

With firms, $N = 2$. $q = \frac{90}{2+1} = 30$

In equilibrium each firm sets $q = \frac{90}{N+1}$. Market quantity $Nq = \frac{N}{N+1}90$.
Price in equilibrium is:

$$q = \frac{90}{N+1}$$

$$Q = Nq = \frac{N}{N+1}90$$

$$p = 100 - \left(\frac{N}{N+1}90\right)$$

1	45.	45.	55.
2	30.	60.	40.
5	15.	75.	25.
10	8.18182	81.8182	18.1818
100	0.891089	89.1089	10.8911
1000	0.0899101	89.9101	10.0899
10000000	≈0	≈ 90.	≈ 10.

2.6 Can we apply symmetry to the profit function?

NO. Each firm best responds to the quantities of the others, they don't get to dictate the quantity of the others. If we apply symmetry first, we create a problem where the firm gets to dictate that others will behave like them.

If we did this we would solve:

$$\pi_i\left(q_i, Q_{-i}\right) = q\left(100 - (Nq)\right) - 10q$$

This is essentially the monopolist problem.