1 Budget Line and Budget Set

Budget Set: B

A competitive budget set are the bundles that a consumer with income k can afford when the prices of the goods are p_1 and p_2.

A bundle is (x_1, x_2). An amount of good 1 and and amount of good 2. What bundles are affordable?

Budget set are all the bundles that meet this affordability condition:

$$p_1x_1 + p_2x_2 \leq m$$

While the budget set are all the affordable bundles, the budget line are all the bundles that are "just affordable" (spend all income).

$$p_1x_1 + p_2x_2 = m$$

Suppose $p_1 = 5$ and $p_2 = 5$ and income $m = 100$

For instance, the bundle $(0, 0)$ is affordable. The bundle $(20, 0)$ is affordable. And the bundle $(10, 10)$ is affordable. The budget line for this example is:

$$5x_1 + 5x_2 = 100$$

$$5x_2 = 100 - 5x_1$$

$$x_2 = \frac{100}{5} - \frac{5x_1}{5}$$

$$x_2 = -x_1 + 20$$

The slope of this line is -1 and the x_2 intercept is 20.

The x_2 intercept is the bundle I can have if I **only** spend money on x_2. In this case $(0, 20)$

The x_1 intercept is the bundle I can have if I **only** spend money on x_1. In this case $(20, 0)$

Slope of the line is -1.

The slope of the budget line measures: *How much x_2 do I have to give up to get one more unit of x_1?*

The slope of the budget line will always be $-\frac{p_1}{p_2}$

$$-\frac{1}{1} = -1$$

$$p_1x_1 + p_2x_2 = m$$

$$p_2x_2 = m - p_1x_1$$

$$x_2 = -\frac{p_1}{p_2}x_1 + \frac{m}{p_2}$$

You have to give up $\frac{p_1}{p_2}$ units of x_2 to get one more unit of x_1 on the budget line.

Suppose $p_2 = 2$ and $p_1 = 1$. If I buy one more unit of x_1, I spend 1 more dollars.

What's the x_2 intercept? The bundle I can have if I only spend money on x_2.

$$\left(0, \frac{m}{p_2}\right)$$

The expression for the x_1 intercept.

$$\left(\frac{m}{p_1}, 0\right)$$

As an asside, suppose I spend half of my income on x_1 and half on x_2. Here is the expression for that bundle:

$$\left(\frac{\frac{1}{2}m}{p_1}, \frac{\frac{1}{2}m}{p_2}\right)$$

1.1 Taxes and other scenarios

$$p_1x_1 + p_2x_2 = m$$

$$p_1x_1 + p_2x_2 = m\,(0.8)$$

Quantity tax on x_1. For every unit of x_1 you buy, you pay t extra to the government. (Gasoline uses a quantity tax).

$$p_1x_1 + (tx_1) + p_2x_2 = m$$

$$(p_1 + t)\,x_1 + p_2x_2 = m$$

Ad valorem tax. For every dollar you spend on x_1, you owe t to the government.

$$(1+t)(p_1x_1) + p_2x_2 = m$$

If we impose an ad valorem tax of t on **both goods**.

$$(1+t)(p_1x_1) + (1+t)\,p_2x_2 = m$$

$$(1+t)\,p_1x_1 + (1+t)\,p_2x_2 = m$$

Slope:

$$-\frac{(1+t)\,p_1}{(1+t)\,p_2} = -\frac{p_1}{p_2}$$

2 The Preference Relation $\succsim$

Introductory course: x is chosen over y because x gives the consumer more **utility** than y.

Utility isn't something that exists. Not the foundation of economics.

Utility is a mathematical convenience for economists.

2.1 What's a Relation

A relation is an abstract mathematical concept.

A relation is a statement about pairs of things.

Example of a relation: "Is at least as tall as" on the set of people.

Shaq is at least as tall as Greg

Greg is not at least as tall as Shaq

Greg is at least as tall as Greg

Shaq is at least as tall as Shaq

Example of a relation: "Is a sibling of" on the set of people.

Shaq is not a brother of Greg

Greg is not a brother of Shaq

Greg is a brother of Christina

Christina is not a brother of Greg

...

Formally a relation $\succsim$ is a subset of the set of order pairs of a set.

$$\succsim \subseteq X \times X$$

If the relation is true for a pair (x, y) we write $x \succsim y$.

$$Shaq \succsim Greg$$

A relation is the abstract mathematical representation of relationships between things.

Some familiar relations:

$\geq$ on the set of numbers. $5 \geq 3$

$<$ on the set of numbers. $3 < 5$

"Sibling of" on the set of people.

"Is as tall as" on the set of people.

"is a friend of" on the set of people

....

In economics we use relations to represent preferences.

A preference relation $\succsim$ on the set of X. We interpret it as "Likes at least as much as."

Suppose Finn is buying ice cream. $(1, 0)$ is one scoop of vanilla and zero scoops of chocolate. Suppose finn likes more ice cream to less, and otherwise prefers vanilla to chocolate.

Suppose the set X is $\{(1, 0), (0, 1), (0, 0), (1, 1)\}$

Finn prefers $(1, 1)$ to $(0, 0)$:

Because he wants more ice cream:

$$(1, 1) \succsim (0, 0)$$

$$(0, 1) \succsim (0, 0)$$

$$(1, 0) \succsim (0, 0)$$

$$(1, 1) \succsim (1, 0)$$

$$(1,1) \succsim (0,1)$$

Because he likes vanilla over chocolate:

$$(1,0) \succsim (0,1)$$

$$(0,0) \succsim (0,0)$$

Notice with the weak relation, we can express both strong (strict) preferences and indifference.

If finn didn't care about flavor, both of these would be true:

$$(1,0) \succsim (0,1)$$

$$(0,1) \succsim (1,0)$$

As short hand, we write $(1,0) \sim (0,1)$ when he is indifference.

We can also express strict preference. For instance.

$$(1,0) \succsim (0,0)$$

$$NOT\ (0,0) \succsim (1,0)$$

When $x \succsim y$ and not $y \succsim x$ we write $x \succ y$ and we say x is **strictly preferred** to y.

When $x \succsim y$ and $y \succsim x$ we write $x \sim y$ and we say x is **indifferent** to y.

2.2 The Preference Relation

2.3 Assumptions on $\succsim$

Axiom 1. Reflexive: For all bundles, the bundle is at least as good as itself.

Axiom 2. Complete: For every pair of distinct bundles, either one is at least as good as the other or the consumer is indifferent.

Axiom 3. Transitivity: If bundle A is preferred to B and bundle B is preferred to C, then bundle A is preferred to C.