1 Preference Types Continued

Perfect Substitutes

$U(x_1, x_2) = ax_1 + bx_2$

Straight-line indifference curves. Always the same willingness to trade-off between x_1 and x_2 (constant MRS for all bundles).

Perfect Complements

$U(x_1, x_2) = min\{ax_1, bx_2\}$

Preference where the consumer is not willing to trade-off between the goods. Consumes x_1 and x_2 in some fixed propositions. L-shaped indiffernce curves.

Examples:

Left and right shoes. $min\{x_1, x_2\}$.

Pies are 2 apples and 1 crust. $U(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$. Kinks are along the line $\frac{1}{2}x_1 = x_2$ This is the "line of no waste" that connects all "no waste points".

Cobb Douglass

$x_1^{\alpha}x_2^{\beta}$. The consumer gets tired of either good if they have too much of it relative to the other good.

1.1 Quasi-Linear Preferences

This is a sort of hybrid between cobb douglass and perfect substitutes.

Example: *Ice cream and other food.*

$U(x_1, x_2) = ln(x_1) + x_2$

$$MRS = -\frac{MU_1}{MU_2} = -\frac{\frac{\partial(ln(x_1)+x_2)}{\partial x_1}}{\frac{\partial(ln(x_1)+x_2)}{\partial x_2}} = -\frac{\frac{1}{x_1}}{1} = -\frac{1}{x_1}$$

MRS (willingness to trade off) only depends on the amount of x_1 I have but not the amount of x_2.

2 Other Preference Assumption

Rational preferences are complete and Transitive Preferences. Most rational preferences can be represented by a utility function.

Lexicographic Preferences cannot be represented by a utility function.

Prefernce over cars is I want more horsepower (x_1) and otherwise if two cars have the same horsepower, I want more trunk space (x_2)

$$(200, 5) \succ (100, 100)$$

$$(100, 100) \succ (100, 5)$$

This is a reasonable and rational preference relation that cannot be represented by a utility function.

2.1 Well-Behaved Preferences.

These assumptions are not necessary to do economics, but they are convenient.

2.2 Monotonicity

More of either good is better.

Either of these assumptions will ensure that the optimal bundle for a consumer must be on the be on the budget line.

2.2.1 Weak Monotonicity (Monotonicity)

Preferences are weakly monotonic (monotonic)

Take two bundles (x_1, x_2) and (y_1, y_2)

1. If $x_1 \geq y_1$ and $x_2 \geq y_2$ then $(x_1, x_2) \succsim (y_1, y_2)$

If 1 is true and...

2. If both $x_1 > y_1$ and $x_2 > y_2$ then $(x_1, x_2) \succ (y_1, y_2)$

Suppose preference are monotonic.

$$(1, 1) \succsim (1, 0)$$

$$(1, 1) \succ (0, 0)$$

Under weak monotonicity, it is possible to have $(1, 1) \sim (1, 0)$

Weak monotonicity is that I want more of both goods and if I get strictly more of both I am strictly better off.

2.2.2 Strict Monotonicity

1. If $x_1 \geq y_1$ and $x_2 \geq y_2$ then $(x_1, x_2) \succsim (y_1, y_2)$

If 1 is true and:

2. If either $x_1 > y_1$ or $x_2 > y_2$ or both then $(x_1, x_2) \succ (y_1, y_2)$

$$(1, 1) \succ (1, 0)$$

$$(1, 1) \succ (0, 0)$$

We cannot have $(1, 1) \sim (1, 0)$.

Strict monotonicity is that I want more of both goods and if I get strictly more of either I am strictly better off.

Perfect complements are weakly but not strictly monotonic.

With $Min\{x_1, x_2\}$. $(1, 1) \sim (2, 1)$

Perfect substitutes are strictly monotonic.

$ax_1 + bx_2$. Increase either x_1 or x_2 and utility strictly increases.

Perfect Complements - Weakly Monotonic

Cobb Douglass, Quasi-Linear, Perfect Substitutes- Strictly Monotonic

2.3 Convexity

Assumption(s) that "mixtures" are better than extremes.

$(10, 0)$ and $(0, 10)$ (extreme bundles).

$(5, 5)$ **is a mixture. Convexity requires I like this mixture better than the worst of the two endpoints.**

2.3.1 Weak Convexity (Convexity)

If there are two bundles (x_1, x_2) and (y_1, y_2) that are indifferent, $(x_1, x_2) \sim (y_1, y_2)$ then any **mixture** (z_1, z_2) must be at least good as either.

$$(10, 0) \sim (0, 10)$$

$$(5, 5) \succsim (10, 0)$$

$$(5, 5) \succsim (0, 10)$$

The possible mixtures are "Convex Combinations". All the points on a straight line between the endpoints.

The possible convex combinations are:

$t \in [0, 1]$

$$(tx_1 + (1 - t)\, y_1, tx_2 + (1 - t)\, y_2)$$

Suppose we have $(10, 0)$ and $(0, 10)$ and use $t = \frac{1}{2}$.

$$\left(\frac{1}{2}(10)+\frac{1}{2}(0),\frac{1}{2}(0)+\frac{1}{2}(10)\right)=(5,5)$$

Formally:

For two bundles $(x_1,x_2)\sim(y_1,y_2)$

For any $t\in[0,1]$

$$(tx_1+(1-t)\,y_1,tx_2+(1-t)\,y_2)\succsim(x_1,x_2)$$

$$(tx_1+(1-t)\,y_1,tx_2+(1-t)\,y_2)\succsim(y_1,y_2)$$

For weakly convex (convex) preferences, if you draw a line between two points on an indifference curve, that line has to lie weakly above the indifference curve.

Examples: Perfect substitutes, Perfect Complements.

2.3.2 Strict Convexity

For two indifferent bundles $(x_1,x_2)\sim(y_1,y_2)$ any mixture is strictly better than the end points.

For any $t\in(0,1)$

$$(tx_1+(1-t)\,y_1,tx_2+(1-t)\,y_2)\succ(x_1,x_2)$$

$$(tx_1+(1-t)\,y_1,tx_2+(1-t)\,y_2)\succ(y_1,y_2)$$

For strictly convex preference, if you draw a line between two points on an indifference curve, that line has to lie strictly above the indifference curve.

Examples: cobb douglass, quasi-linear.

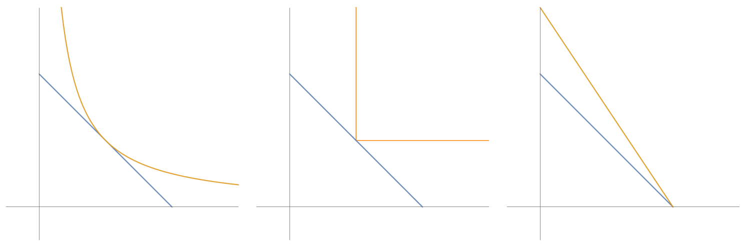
3 Choice

We define choice as:

A consumer will choose a bundle from the budget that is at least as good as everything else in the budget set.

3.1 Optimal Point Cannot be on an Indifference Curve that Crosses into Budget Set.

3.2 Three Possibilities



3.2.1 Just Touching (Tangent)

3.2.2 Just Touching (Kinked)

3.2.3 Just Touching (Boundary)

3.3 Examples

3.3.1 Cobb-Douglas:

$u(x_1, x_2) = x_1 x_2, p_1 = 1, p_2 = 1, m = 10.$

3.3.2 Perfect Substitutes

$u(x_1, x_2) = 2x_1 + x_2$ with p_1 =1, $p_2 = 1, m = 10$.

3.3.3 Perfect Complements (Left and Right Shoes)

$u(x_1, x_2) = min\{x_1, x_2\}$ and suppose $p_1 = 2$,$p_2 = 1$,$m = 15$.

3.3.4 Perfect Complements (2 Apples, 1 Crust)

$u(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$ and suppose $p_1 = 2$,$p_2 = 1$,$m = 15$.