1 Optimal Points Cannot be on Indifference Curves that Pass Into Budget Set

1.1 Three Possibilities

2 Examples

2.1 Cobb Douglass

$u(x_1, x_2) = x_1 x_2$, $p_1 = 1, p_2 = 1, m = 10$

Because prefrences are monotonic. The optimal bundle will occur on the budget line.

Budget Condition.

$p_1 x_1 + p_2 x_2 = m$

$1x_1 + 1x_2 = 10$.

"Tangency Condition"

$$-\frac{1}{1} = MRS$$

$$-\frac{1}{1} = -\frac{\frac{\partial(x_1 x_2)}{\partial x_1}}{\frac{\partial(x_1 x_2)}{\partial x_2}}$$

$$-\frac{1}{1} = -\frac{x_2}{x_1}$$

$$x_1 = x_2$$

Let's solve these two conditions:

Budget:

$$1x_1 + 1x_2 = 10$$

Tangency:

$$x_1 = x_2$$

Plug the tangency condition into the budget condition:

$$1x_1 + 1x_1 = 10$$

$$2x_1 = 10$$

$$x_1 = 5$$

Plugging this back into the tangency condition (or the budget) gives us:

$$x_2 = 5$$

The optimal bundle is:

$$(5, 5)$$

2.2 Cobb Douglass General

$u(x_1, x_2) = x_1 x_2$. Budget: $p_1 x_1 + p_2 x_2 = m$
Budget Condition:

$$p_1 x_1 + p_2 x_2 = m$$

Tangency Condition:

$$-\frac{p_1}{p_2} = -\frac{x_2}{x_1}$$

Simplified to:

$$x_1 p_1 = x_2 p_2$$

Let's solve these two equations:

$$p_1 x_1 + p_2 x_2 = m$$

Plug in the tangency condition:

$$p_1 x_1 + p_1 x_1 = m$$

$$2p_1 x_1 = m$$

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

Plug this back into the tangency condition:

$$p_1 \frac{\frac{1}{2}m}{p_1} = p_2 x_2$$

$$\frac{1}{2}m = p_2 x_2$$

$$x_2 = \frac{\frac{1}{2}m}{p_2}$$

This equation says "spend the same amount on both goods".

$$x_1 = \frac{\frac{1}{2}m}{p_1}, x_2 = \frac{\frac{1}{2}m}{p_2}$$

2.3 Perfect Complements #1

$u(x_1, x_2) = min\{x_1, x_2\}$, $p_1 = 1, p_2 = 1, m = 10$
"No Waste Condition"

$$x_1 = x_2$$

Budget Condition.

$$x_1 + x_2 = 10$$

Let's solve these by plugging the no-waste condition into the budget equation

$$x_1 + x_2 = 10$$

Plug in the $x_1 = x_2$

$$x_1 + x_1 = 10$$

$$2x_1 = 10$$

$$x_1 = 5$$

From $x_1 = x_2$:

$$x_2 = 5$$

$$(5, 5)$$

$min\{x_1, x_2\}, p_1 x_1 + p_2 x_2 = m$

$$x_1 = \frac{m}{p_1 + p_2}, x_2 = \frac{m}{p_1 + p_2}$$

2.4 Perfect Complements #2

$u(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$, $p_1 = 1, p_2 = 1, m = 15$
No Waste Condition, Budget Equation:

$$\frac{1}{2}x_1 = x_2, x_1 + x_2 = 15$$

$$x_1 = 10, x_2 = 5$$

$$10 * 1 + 5 * 1 = 15$$

2.5 Perfect Complements #2

$u(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$, $p_1x_1 + p_2x_2 = m$

$$\frac{1}{2}x_1 = x_2, p_1x_1 + p_2x_2 = m$$

$$p_1x_1 + p_2x_2 = m$$

$$p_1x_1 + p_2\left(\frac{1}{2}x_1\right) = m$$

$$x_1 = \frac{m}{p_1 + \frac{1}{2}p_2} = \frac{2}{2}\frac{m}{p_1 + \frac{1}{2}p_2} = 2\frac{m}{2p_1 + p_2}$$

$$\frac{1}{2}x_1 = x_2$$

$$\frac{1}{2}\left(2\frac{m}{2p_1 + p_2}\right) = x_2$$

$$\frac{m}{2p_1 + p_2} = x_2$$

2.6 Perfect Substitutes

$u(x_1, x_2) = 2x_1 + x_2,\ p_1 = 1, p_2 = 1, m = 10$

$$MRS = -\frac{2}{1}$$

$$-\frac{p_1}{p_2} = -1$$

Since I am always willing to give up two units of x_2 to get one of x_1 but I only have to give up one unit (slope of the budget equation). I should always do it. Always give up a unit of good 2 to get one of good 1.

With perfect substitutes, if the indifference curve is steeper than the budget line buy only good 1.

With perfect substitutes, if the indifference curve is shallower than the budget line buy only good 2.

Greg's Solution:

1. Find the intercepts.

$$(10, 0), (0, 10)$$

2. Calculate the utility of these bundles.

$$u(10, 0) = 2(10) + 0 = 20$$

$$u(0, 10) = 2(0) + 10 = 10$$

Suppose we had $u(x_1, x_2) = x_1 + x_2$

$$u(10, 0) = 10$$

$$u(0, 10) = 10$$

In this case, any bundle that costs m is optimal.

2.7 Quasi-Linear Example

$u(x_1, x_2) = ln(x_1) + x_2, p_1 = 1, p_2 = 1, m = 10$

Budget:

$$x_1 + x_2 = 10$$

Tangency:

$$-\frac{1}{1} = MRS$$

$$-\frac{1}{1} = -\frac{\frac{\partial(ln(x_1)+x_2)}{\partial x_1}}{\frac{\partial(ln(x_1)+x_2)}{\partial x_2}}$$

$$-\frac{1}{1} = -\frac{\frac{1}{x_1}}{1}$$

$$x_1 = 1$$

Plug this back into the budget equation to get x_2.

$$x_2 = m - 1$$