Comparative statics, how does the solution to the model change when we change just one of the parameters such as one of the prices.

1 Intertemporal Choice

How a consumer chooses how much they will consume today vs in the future. Borrow and saving behavior.

1.1 Bundles (Consumption Today, Consumption Tomorrow)

(c_1, c_2)

c_1 is the amount of money I'll spend today.

c_2 is the amount of money I'll spend in the future.

Endowment is the income of the consumer today m_1 and the income in the future m_2.

Retirement savings.

$m_1 = 1,000,000$, $m_2 = 0$

$u = min\{c_1, c_2\}$

Income grows over time:

$m_1 = 10000$, $m_2 = 20000$

1.2 Budget Constraint

Suppose all I can do is save money for tomorrow by putting it in a piggy bank.

$$c_2 = m_2 + (m_1 - c_1)$$

My consumption in period 2 is the income income in period 2 (m_2) plus whatever I save in period 1 ($m_1 - c_1$)

$$c_1 + c_2 = m_1 + m_2$$

Here, I can borrow or save at zero interest. If the interest rate wasn't zero and I had to pay back more than what borrow, this bundle wouldn't be possible:

We are going to assume that money can borrowed or saved with an interest rate of r.

If I borrow an amount b today, I have to pay back $b + rb$ tomorrow.

For instance, if I borrow 1000 and $r = 0.1$. Then the amount I pay back in the next period is $1000 + 1000 * 0.1 = 1100$

If I save an amount s then tomorrow I get back $s + rs$.

For instance, if I save 1000 and $r = 0.1$. Then the amount I get back in the next period is $1000 + 1000 * 0.1 = 1100$

Suppose the consumer saved money in period one. They saved $(m_1 - c_1)$.

$$c_2 = m_2 + (m_1 - c_1) + r(m_1 - c_1)$$

$$c_2 = m_2 + (1+r)(m_1 - c_1)$$

Suppose the consumer borrowed money in period one. They borrowed $(c_1 - m_1)$.

$$c_2 = m_2 - (c_1 - m_1) - r(c_1 - m_1)$$

$$c_2 = m_2 - (1+r)(c_1 - m_1)$$

Rearranging this:

$$c_2 = m_2 + (1+r)(m_1 - c_1)$$

$$(1+r)c_1 + c_2 = (1+r)m_1 + m_2$$

1.3 Plotting the Budget Equation

The c_1 intercept is $(c_1, 0)$

$$(1+r)c_1 + 0 = (1+r)m_1 + m_2$$

$$c_1 = \frac{(1+r)m_1 + m_2}{(1+r)} = m_1 + \frac{m_2}{1+r}$$

$\frac{m_2}{1+r}$ is the biggest loan I could take. $(1+r)\frac{m_2}{1+r} = m_2$

1.4 Two Versions of the budget equation

Future value versions of the budget equation

$$(1+r)c_1 + c_2 = (1+r)m_1 + m_2$$

Present value versions of the budget equation

$$c_1 + \frac{c_2}{(1+r)} = m_1 + \frac{m_2}{(1+r)}$$

1.5 Example Problem

Suppose $m_1 = 200$, $m_2 = 600$, and $r = \frac{1}{2}$. Utility is: $u(c_1, c_2) = c_1 c_2$.

$$c_2 = m_2 + (1+r)(m_1 - c_1)$$

$$(1+r)c_1 + c_2 = (1+r)m_1 + m_2$$

$$c_1 + \frac{c_2}{1+r} = m_1 + \frac{m_2}{1+r}$$

$$MRS = -(1+r)$$

$$-\frac{c_2}{c_1} = -1.5$$

$$c_2 = 1.5c_1$$

Plug this into the budget constraint:

$$(1+r)c_1 + c_2 = (1+r)m_1 + m_2$$

$$1.5c_1 + c_2 = 1.5(200) + 600$$

$$1.5c_1 + c_2 = 900.$$

$$c_2 + c_2 = 900$$

$$c_2 = 450$$

$$c_1 = \frac{450}{1.5} = 300.$$

$$(300, 450)$$

1.6 Comparative Statics