1 Utility

When $\succsim$ is reflexive, complete, transitive, we can put any set of bundles into an "order" potentially with ties.

Example: Suppose $\succsim$ is such that $a \succ b, a \succ c, b \succ c$.

Our ordering:

3- a

2- b

1- c

$u(a) = 3, u(b) = 2, u(c) = 1$

This utility function represents this preference relation. We can us it in place of the preference relation, because it summarizes everything we need to know.

$a \succ b$, $u(a) > u(b)$

Example: Suppose $\succsim$ is such that $a \sim b, a \succ c, b \succ c$.

a,b

c

$u(a) = 2, u(b) = 2, u(c) = 1$

$u(a) = 4, u(b) = 4, u(c) = 2$

1.1 Utility Defined

Utility function is a "mapping" from bundles into numbers.

$U : X \to \mathbb{R}$

U represents $\succsim$ when:

If $x \succsim y$ then $U(x) \geq U(y)$

If $U(x) \geq U(y)$ then $x \succsim y$

1.2 Example

Example: Bowls of ice cream (x_1, x_2) where x_1 is scoops of vanilla and x_2 is scoops of chocolate.

Consumer wants more ice cream, doesn't care about flavor.

$(1,1) \succ (0,1)$

Define the utility of a bowl to be the total scoops of ice cream:

$U(x_1, x_2) = x_1 + x_2$

$U(1,1) = 2$, $U(0,1) = 1$

1.3 Increasing Transformations

$U(x_1, x_2) = (x_1 + x_2) + 10$

$U(1,1) = 12, U(0,1) = 11, U(1,0) = 11$

$U(x_1, x_2) = (x_1 + x_2)^2$

$U(1,1) = 4, U(0,1) = 1, U(1,0) = 1$

Increasing transformations:

Adding numbers, squaring, multiplying by a number, **taking the log...**

1.4 Marginal Rate of Substitution from Utility Function

The slope of the indifference curve also called the **Marginal Rate of Substitution (MRS)** tells us (roughly) how much of x_2 the consumer would give up to get one more unit of x_1.

$U(x_1, x_2) = x_1 + x_2$

An indifference is a set of bundles that all give the same utility. The equation for all of the bundles that give this consumer utility of 1 is:

$$x_1 + x_2 = 1$$

The equation for all of the bundles that give this consumer utility of 2 is:

$$x_1 + x_2 = 2$$

$$MRS = -\frac{\frac{\partial(U(x_1,x_2))}{\partial x_1}}{\frac{\partial(U(x_1,x_2))}{\partial x_2}}$$

Marginal utility:

$$\frac{\partial(U(x_1, x_2))}{\partial x_1} = MU_1$$

How much does utility increase if we increase x_1 while holding x_2 fixed.

$$MRS = -\frac{MU_1}{MU_2}$$

$$MU_1 = \frac{\partial(x_1 + x_2)}{\partial x_1} = 1$$

$$MU_2 = \frac{\partial(x_1 + x_2)}{\partial x_2} = 1$$

$$MRS = -\frac{1}{1} = -1$$

Suppose we had:
$MU_1 = 1$ and $MU_2 = 2$

1.5 Derivative Notes

$$\frac{\partial\left(x_1 + x_2\right)}{\partial x_1} = 1$$

$$\frac{\partial\left(x_1 x_2\right)}{\partial x_1} = x_2$$

$$\frac{\partial\left(x_1 x_2 + x_2\right)}{\partial x_1} = x_2$$

$$5x_1 + 5 = 5$$

$$\frac{\partial\left(x_1 x_2 + x_2\right)}{\partial x_2} = x_1 + 1$$

2 Some Examples of Preferences and Utility

2.1 Prefect Substitutes

The rate a consumer is willing to trade-off between the two goods is the same for at bundle. The consumer's willingness to trade-off does not depend on what bundle they have.
Only cares about total scoops:

$$U\left(x_1, x_2\right) = x_1 + x_2$$

Like vanilla two-times more than chocolate.

$$U\left(x_1, x_2\right) = 2x_1 + x_2$$

$$-\frac{\frac{\partial(2x_1 + x_2)}{\partial x_1}}{\frac{\partial(2x_1 + x_2)}{\partial x_2}} = -\frac{2}{1} = -2$$

If I like x_2 better:

$$U(x_1, x_2) = 2x_1 + 5x_2$$

$$-\frac{\frac{\partial(2x_1+5x_2)}{\partial x_1}}{\frac{\partial(2x_1+5x_2)}{\partial x_2}} = -\frac{2}{5}$$

2.2 Cobb Douglass

The consumer gets tired of either good if they have too much of it.

$$U(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$$

For example: $\alpha = \beta = 1$

$$U(x_1, x_2) = x_1 x_2$$

$$MRS = -\frac{\frac{\partial(x_1 x_2)}{\partial x_1}}{\frac{\partial(x_1 x_2)}{\partial x_2}} = -$$

$$\frac{\partial(x_1 x_2)}{\partial x_1} = x_2$$

$$\frac{\partial(x_1 x_2)}{\partial x_2} = x_1$$

$$MRS = -\frac{x_2}{x_1}$$

$MRS(10, 1) = -\frac{1}{10}$
$MRS(1, 10) = -\frac{10}{1} = -10$

$$MRS(5, 5) = -\frac{5}{5} = -1$$

If two utility functions have the same MRS at every point, they represent the same preferences.

$$U(x_1, x_2) = 5\left((x_1 + x_2)^2\right) + 10$$

$$MRS = -\frac{\frac{\partial\left(5\left((x_1+x_2)^2\right)+10\right)}{\partial x_1}}{\frac{\partial\left(5\left((x_1+x_2)^2\right)+10\right)}{\partial x_2}} = -1$$

$$U(x_1, x_2) = x_1 + x_2$$

$$MRS = -\frac{\frac{\partial(x_1+x_2)}{\partial x_1}}{\frac{\partial(x_1+x_2)}{\partial x_2}} = -1$$

2.3 Prefect Complements

You are never willing to trade-off between the goods. The goods are consumed in **fixed proportions**.
Left/Right shoes.

$$U(x_1, x_2) = Min\{x_1, x_2\}$$

Two useable pairs of shoes:

$$(2, 2)$$

Not willing to give up and right shoes to get one left shoe.
Two apples and one crust for every pie.

$$(2, 1)$$

Not willing to give up any crusts to get one more apple.

$$Min\left\{\frac{1}{2}x_1, x_2\right\}$$

2.4 Quasi-Linear

3 Other Preference Assumptions

3.1 Monotonicity

3.1.1 (Weak) Monotonicity

3.1.2 Strict Monotonicity

3.2 Convexity

3.2.1 (Weak) Convexity

3.2.2 Strict Convexity