1 The Preference Relation $\succsim$

1.1 What is a relation?

A **relation** is an abstract mathematical concept that tells about the relationship of two "things".

1.2 Examples

A relation always refers to objects in **some set**. We have to designate which set we are talking about. For preferences that set is X (the feasible set) but for other relations is it some other set. Here are some examples.

$\geq$ is the "greater than or equal to" relationship on the **set of numbers**.

$$5 \geq 4$$

$$3 \geq 2$$

S is the relation "sibling of" on the set of people.

$$Greg\, \boldsymbol{S}\, Christina$$

$$Christina\, \boldsymbol{S}\, Greg$$

B is the relation "taller than" on the set of people.

$$Greg\, \boldsymbol{S}\, Christina$$

$$Shaq\, \boldsymbol{S}\, Greg$$

1.3 Preference Relation

$\succsim$ "at least as good as relation" on the set X (the feasible set).

When $x \succsim y$ we say "x is preferred to y"

Let's look at our ice cream example. Suppose Finn likes more ice cream to less ice cream but otherwise does not care about flavor. Here are some things true about Finn's preferences.

$$(1,0) \succsim (0,0)$$

$$(0,1) \succsim (0,0)$$

$$(1,1) \succsim (0,1)$$

$$(2,2) \succsim (1,1)$$

$$(1,1) \succsim (1,1)$$

$$(0,0) \succsim (0,0)$$

1.4 Indifference and Strict Preference

1.4.1 Indifference

What about $(1,0)$ and $(0,1)$? Since they have the same amount of ice cream and Finn does not care about flavor, he is **indifferent** between them.

$$(1,0) \succsim (0,1)$$

$$(0,1) \succsim (1,0)$$

In this case, we can also write:

$$(1,0) \sim (0,1)$$

1.4.2 Strict Preference

On the other hand:

$$(2,0) \succsim (1,0)$$

$$(1,0) \not\succsim (2,0)$$

In this case, Finn strictly prefers $(2,0)$ to $(1,0)$. We can write:

$$(2,0) \succ (1,0)$$

Note how $\succsim$ allows us to express both $\sim$ (indifference) and $\succ$ (strict preference).

1.5 Assumptions

The **Axioms** of preferences.

1.5.1 Reflexivity

For all bundles $x \in X$.

$$x \succsim x$$

This is a **technical** assumption that is very uncontroversial. It says they **every bundle is indifferent to itself**.

1.5.2 Completeness

For all pairs $x, y \in X$.

Either $x \succsim y$ or $y \succsim x$ or both.

For every pair, you have to have something to say. You can strictly prefer one to the other or be indifferent between them.

This rules out someone having no idea how the compare the bundles.

1.5.3 Transitivity

For all $x, y, z \in X$

If $x \succsim y$ and $y \succsim z$ then $x \succsim z$.

Here is an example that does not violate transitivity:

$$(2,0) \succ (1,0)$$

$$(1,0) \succ (0,0)$$

$$(2,0) \succ (0,0)$$

Notice that these bundles have an "order" $(2,0), (1,0), (0,0)$. Higher bundles in this order are preferred to lower bundles in this order.

If we don't have transitivity, we can't always put bundles in an order.

Here is a violation of transitivity.

$$(2,0) \succ (0,2)$$

$$(0,2) \succ (1,1)$$

$$(1,1) \succ (2,0)$$

To see that this violates transitivity note that:

Since $(2,0) \succ (0,2)$ we also know $(2,0) \succsim (0,2)$

Since $(0,2) \succ (1,1)$ we also know that $(0,2) \succsim (1,1)$

We have $(2,0) \succsim (0,2)$ and $(0,2) \succsim (1,1)$ by transitivity it would have to be that $(2,0) \succsim (1,1)$. However, Finn's preferences are $(1,1) \succ (2,0)$ this implies $(2,0) \not\succsim (1,1)$. Thus, we have a violation of transitivity.

Let's try to put $\{(2,0),(0,2),(1,1)\}$ in order of Finn's preferences.

$$(2,0),(0,2),(1,1),(2,0)$$

This is not an order, it is more like a cycle. When preferences are not transitive (intransitive) there may be no "best" bundle from a set. Here, Finn has no favorite bundle from the set $B = \{(2,0),(0,2),(1,1)\}$. He cannot make a choice from this set. **For any bundle there is always something strictly better.** This would be problematic for Finn making a choice. Since we are trying to model choice, intransitivity is going to cause lots of problems.

Transitivity is what ensures that there is always some choice from any budget set, because it allows us to put bundles in "order".

1.6 Rational Preferences

We say $\succsim$ is "rational" if it meets these three assumption.

Rational preferences can be "weird" but they will still work for our framework of choice. **Rationality puts very few restrictions on preferences.**

1.7 Utility From Preferences

Let's assign a "preference score" to the bundles where a bundle ranked higher gets a higher score. Let's use $u()$ to represent the score. We will return to Finn liking more ice cream to less:

$$u(2,0) = 3$$

$$u(1,0) = 2$$

$$u(0,0) = 1$$

These "scores" represent the location of the bundle in the preference order. This is all a utility is. It is a numeric representation of the preference order that is generated by having transitive preferences. Here is another preference score that represents the same ordering.

$$u(2,0) = 30$$

$$u(1,0) = 5$$

$$u(0,0) = 0$$

These are **utility functions** they are convenient *numerical representations of the underlying preference relation.*

1.8 Indifference Curves and Other Sets

Every bundle is associated with some "sets" of bundles.

$\sim(x)$ the set of all bundles that are indifferent to x. The "Indifference Set" of x.

Finn likes more ice cream to less and otherwise does not care about flavor. What is in the set $\sim(1,1)$?

Because $(2,0) \sim (1,1)$, $(2,0)$ is in the "indifference set" of $(1,1)$. Likewise $(0,2)$ is in this set. We say:

$$(2,0) \in \sim (1,1)$$

$$(0,2) \in \sim (1,1)$$

Here are some other bundles in the set:

$$(1,1) \in \sim (1,1)$$

$$\left(\frac{3}{2}, \frac{1}{2}\right) \in \sim (1,1)$$

This set of bundles $(2,0), (1,1), (0,2), \left(\frac{3}{2}, \frac{1}{2}\right)$ .... are all indifferent to each other. We can also refer to this set as an **indifference curve.**

Indifference curve is set of bundles that indifferent to each other. Indifference curves allow us to "visualize" preferences.

Here are some other sets to be aware of:

$\succ(x)$ strictly better than set

$\succsim(x)$ better than set (upper contour set)

$\prec(x)$ strictly worse than set

$\precsim(x)$ worse than set (lower contour set)

1.9 Indifference Curves Cannot Cross