1 Consumer Choice

Goal is to find the best bundle in their budget set B according to their preferences $\succsim$.

$$x^* = \{x | x \in B \,\&\, \forall y \in B, x \succsim y\}$$

The consumer is looking for a bundle that is at least as good as all other bundles in their budget set.

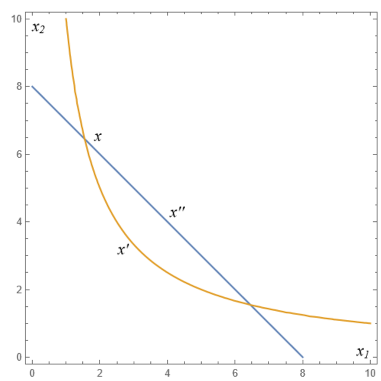
x^* **is the "optimal bundle", consumer's demand.**

1.1 Consumer Problem

The consumer is looking to find a bundle (x_1^*, x_2^*) that maximizes $u(x_1, x_2)$ subject to $p_1x_1 + p_2x_2 \le m$.

1.2 Where to Look for Optimal Bundle

If preferences are monotonic (more is better), then **the optimal bundle will always occur on the budget line**. That is, at a point where $p_1x_1 + p_2x_2 = m$.



Bundle x is on an indifference curve that passes "inside" (interior) of the budget set.

Bundle x is indifferent to x' because they are on the same indifference curve.

$$x \sim x'$$

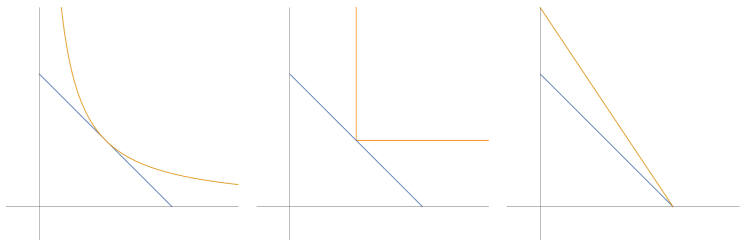
However, the bundle x' is on the interior of the budget set so it cannot possible be optimal. In fact, there must be a bundle like x'' that has strictly more of everything that x' and thus, by monotonicity, $x'' \succ x' \sim x$.

A bundle that is on an indifference curve that crosses into the interior of the budget set cannot be optimal.

First, the optimal bundle will always occur on the budget line. Second, a bundle that is on an indifference curve that crosses into the interior of the budget set cannot be optimal.

Any optimal bundle, must be on the budget line and on an indifference curve that "just touches" but does not cross into the interior of the budget set.

1.3 Three Possibilities



If the utility function is "smooth" (we can take derivatives of it), the optimal bundle can occur at a "tangency" point.

1) If a utility function is smooth, look for a tangency point.

2) In the case of perfect substitutes, even though the utility is smooth and the slope of the indifference is well-defined, the slope of the indifference curve might never be equal to the slope of the budget line. In this case, the optimal bundle must occur at a "corner".

3) For a non-smooth utility function (perfect complements) the optimal bundle must occur at a kink point.

1.4 Examples

1.4.1 Cobb-Douglas One

Let's solve $u(x_1, x_2) = x_1 x_2$, $p_1 = 2, p_2 = 1, m = 20$.

Since the slope of the indifferences (MRS) are well defined, we start by looking for tangency points (where my willingness to trade-off is equal to how I have to trade off), where the slope of the indifference curve is equal to the slope of the utility function.

Tangency Condition:

$$MRS = -\frac{p_1}{p_2}$$

$$-\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

$$x_2 p_2 = x_1 p_1$$

At the optimum, the amount of money spent on both goods is exactly the same.

$$2x_2 = 1x_1$$

Second condition, is the budget condition.

$$x_1 + 2x_2 = 20$$

We have two equations and two unknowns, we can solve these to find the optimal bundle.

$$2x_2 = x_1$$

$$x_1 + 2x_2 = 20$$

Solve this system of equations:

$$2x_2 + 2x_2 = 20$$

$$4x_2 = 20$$

$$x_2 = 5$$

$$2\left(5\right) = x_1$$

$$x_1 = 10$$

Optimal bundle is $(10, 5)$.

1.4.2 Cobb-Douglas Generic Prices and Income

Let's solve $u\left(x_1, x_2\right) = x_1 x_2$, $p_1 x_1 + p_2 x_2 = m$.

Tangency condition:

$$p_1 x_1 = p_2 x_2$$

Budget condition:

$$p_1 x_1 + p_2 x_2 = m$$

Solve this system of equations:

$$\left(\frac{\frac{1}{2}m}{p_1}, \frac{\frac{1}{2}m}{p_2}\right)$$

1.5 More Generic Cobb Douglass

$u\left(x_1, x_2\right)=x_1^\alpha x_2^\beta$, p_1, p_2, m

$$\left(\frac{\frac{\alpha}{\alpha+\beta} m}{p_1}, \frac{\frac{\beta}{\alpha+\beta} m}{p_2}\right)$$

1.5.1 Perfect Substitutes

Let's solve $u\left(x_1, x_2\right)=2 x_1+x_2$ and suppose $p_1=1$,$p_2=1$,$m=10$.

Let's try to find the tangency condition:

$$-\frac{2}{1}=-\frac{1}{1}$$

$$2=1$$

Notice, this can be met. This tells us that the optimal must occur at "corner".

$$(10,0)$$

$$u(10,0)=20$$

$$(0,10)$$

$$u(0,10)=10$$

The optimal bundle is $(10,0)$.

1.5.2 Perfect Substitutes

Let's solve $u\left(x_1, x_2\right)=2 x_1+x_2$ and suppose $p_1=2$,$p_2=1$,$m=10$.

$$u(5,0)=10$$

$$u(0,10)=10$$

Utility of the two corners is the same and so **any bundle** on the budget is optimal.

1.5.3 Perfect Complements (Left and Right Shoes)

Suppose a consumer buys x_1 left shoes and x_2 right shoes. Their utility function is $u(x_1, x_2) = min\{x_1, x_2\}$ and suppose $p_1 = 2$,$p_2 = 1$,$m = 15$.

With perfect complements, there is no tangency condition because we cannot find the slope of the indifference curve. What replaces the tangency condition here is the **no waste condition**.

Two conditions. **1) No Waste**. **2) Budget.**

$$x_1 = x_2$$

$$2x_1 + 1x_2 = 15$$

Solve this system of equations:

$$2x_2 + 1x_2 = 15$$

$$3x_2 = 15$$

$$x_2 = 5$$

$$x_1 = 5$$

$$(5, 5)$$

1.5.4 Perfect Complements (2 Apples, 1 Crust)

Suppose a consumer makes pies using 2 apples x_1 and 1 crust x_2 for every pie. Their utility function is $u(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$ and suppose $p_1 = 2$,$p_2 = 1$,$m = 15$.

No Waste

$$\frac{1}{2}x_1 = x_2$$

Budget

$$2x_1 + x_2 = 15$$

Solve the system of equations:

$$2x_1 + \frac{1}{2}x_1 = 15$$

$$2.5x_1 = 15$$

$$x_1 = 6.$$

$$3 = x_2$$

$$(6, 3)$$