0.1 Slutsky Decomposition Perfect Complements Example

$u(x_1, x_2) = min\{x_1, x_2\}$.

$p_1 = 1$ and $p_2 = 1$. $m = 12$. What is the substitution and income effect on demand for x_1 when p_1 increases to $p_1 = 2$.

Find the demands for x_1 and x_2.

No waste condition:

$$x_1 = x_2$$

Budget condition:

$$p_1x_1 + p_2x_2 = m$$

To find the Marshallian demand, we solve these equations "simultaneously".

$$p_1x_1 + p_2x_1 = m$$

$$x_1(p_1 + p_2) = m$$

Marshallian Demands:

$$x_1 = \frac{m}{p_1 + p_2}, x_2 = \frac{m}{p_1 + p_2}$$

Initial demand $p_1 = 1, p_2 = 1, m = 12$

$$x_1 = \frac{12}{2} = 6, x_2 = \frac{12}{2} = 6$$

$$(6, 6)$$

After the price change: $p_1 = 2, p_2 = 1, m = 12$

$$x_1 = \frac{12}{3} = 4, x_2 = \frac{12}{3} = 4$$

$$(4, 4)$$

Total change: x_1 decreases by 2 units after the price change.

Decompose the 2 unit decrease in demand into the substitution effect and income effect.

We need to construct the "thought experiment budget". Give them enough income to afford their old bundle at the new prices.

Old bundle:

$$(6, 6)$$

The cost of the old bundle at the new prices:

$$2\,(6) + 1\,(6) = 18$$

The consume needs 18 to afford the old bundle at the new prices. The "thought experiment" budget equation:

$$2\,(x_1) + 1\,(x_2) = 18$$

What do they buy?

$$x_1 = \frac{m}{p_1 + p_2}, x_2 = \frac{m}{p_1 + p_2}$$

$$x_1 = \frac{18}{3} = 6, x_2 = \frac{18}{3} = 6$$

$$(6, 6)$$

The original demand for x_1 minus the demand for x_1 in the thought experiment is the substitution effect:

$$6 - 6 = 0$$

The a zero decrease in demand due to the substitution effect. The whole effect must be due to the income effect.

Income effect is the remaining change in demand:

$$2$$

All of the effect is due to income effect.

0.2 Perfect Substitutes Example.

$u\,(x_1, x_2) = x_1 + x_2$. $p_1 = 1, p_2 = 2$, $m = 10$. Then price changes to $p_1 = 3$. How much of the change in demand for x_1 is due to substitution and how much is due to income?

Calculate the utility of the "intercepts" of budget:

$$u\,(10, 0) = 10$$

$$u\,(0, 5) = 5$$

Demand before the price change when $p_1 = 1, p_2 = 2, m = 10$ is.

$$(10, 0)$$

Demand after the price change $p_1 = 3, p_2 = 2, m = 10$:
Compare the utilility of the endpoints:

$$u\left(\frac{10}{3}, 0\right) = 3.33333$$

$$u\left(0, 5\right) = 5$$

$$\left(0, 5\right)$$

The increase in price of good one, decreased demand for good 1 by 10 units. How much of this is due to substitution?

How much income do I need to afford the old bundle $(10, 0)$ at the new prices?

$$3\left(10\right) + 0\left(2\right) = 30$$

"thought experiment budget" $p_1 = 3, p_2 = 2, m = 30$
What does the consumer buy?

$$u\left(10, 0\right) = 10$$

$$u\left(0, 15\right) = 15$$

Buys:

$$\left(0, 15\right)$$

Substitution effect is the initial demand for x_1 minus the demand under the thought experiment:

$$10 - 0 = 10$$

Substitution effect is responsible for the entire change in demand.

0.3 Something to try.

Repeat the last problem but where p_1 increases to $p_1 = 1.5$ rather than 3.

0.4 Substitution Effect is Always Negative

An increase in the price of a good will always lead to either a zero change or a decrease in demand for that good **due to substitution.**

Since substitution has to be negative, we have only scenarios when the price of a good goes up.

Total Change In Demand	Substitution Effect	Income Effect
- (Ordinary)	-	- (Normal)
- (Ordinary)	-	+ (Inferior)
+ (Giffen)	-	+ (Inferior)

Because **substitution effect** will **always lead to a decrease (or zero) change in demand**. To get a **Giffen good**, the good **must be inferior** and so inferior that the **increase in demand due to the income effect** overwhelms the **decrease in demand due to substitution**.

0.5 Example with Given Demands

Before a price change a consumers demand is:

$$(10, 5)$$

After the price of good 1 increases the consumers demand is

$$(15, 5)$$

If the consumer had enough income to buy $(10, 5)$ at the new prices they buy:

$$(5, 5)$$

Good 1 is giffen. The price of p_1 when up and I demand more of it.

Demand decreases by 5 due to substitution.

Demand increases by 10 due to income effect. (inferior good).

1 Buying and Selling

$$p_1x_1 + p_2x_2 = m$$

m (income) is exogenous. Is it determined outside of the model.

We might want to make it endogenous by having it determined within the model.

1.1 Income to Endowments

An endowment is a bundle of good the consumer starts with.

x_1 is apples and x_2 is crusts.

Farmer has endowment of $(10, 0)$ (10 apples).

Baker has endowment of $(0, 5)$ (5 crusts).

The consumer's incomes is no longer a fixed amount of money, instead it is the "market value" of their endowment under the prices prevailing in the model.

Farmers budget:

$$p_1 x_1 + p_2 x_2 \leq p_1 (10) + p_2 (0)$$

$$p_1 x_1 + p_2 x_2 \leq 10 p_1$$