1 Buying and Selling

1.1 Income to Endowments

Exogenous income: m.

Endogenous income (endowment of goods): (ω_1, ω_2).

Endowment is a **bundle of goods** that the consumer starts with.

Model of consumers who eat pies.

Farmer has an endowment of 10 apples and 0 crusts $(\omega_1, \omega_2) = (10, 0)$.

Baker has an endowment of 0 apples and 5 crusts $(\omega_1, \omega_2) = (0, 5)$.

Someone else has an endowment of 10 apples and 5 crusts $(10, 5)$.

To get the budget we imagine a market where consumers can buy or sell goods at prices p_1, p_2. To determine income, we imagine the consumer going to the market and selling their endowment in exhange for money the can then use to buy goods:

Budget:

$$p_1 x_1 + p_2 x_2 \leq p_1 \omega_1 + p_2 \omega_2$$

1.2 Example of Budget From Endowments and Prices

Suppose we have a farmer with $(\omega_1, \omega_2) = (10, 0)$. Prices are $p_1 = 1, p_2 = 2$:

$$1x_1 + 2x_2 \leq 1\,(10) + 2\,(0)$$

$$x_1 + 2x_2 \leq 10$$

Budget line:

$$x_1 + 2x_2 = 10$$

1.3 Gross Demand vs Net Demand

From the budget set: $p_1 x_1 + p_2 x_2 \leq p_1 \omega_1 + p_2 \omega_2$

Budget line: $p_1 x_1 + p_2 x_2 = p_1 \omega_1 + p_2 \omega_2$

Value of the bundle purchased (x_1, x_2) is equal to the value of the endowment.

$$p_1 x_1 - p_1 \omega_1 + p_2 x_2 - p_2 \omega_2 = 0$$

$$p_1\,(x_1 - \omega_1) + p_2\,(x_2 - \omega_2) = 0$$

$(x_1 - \omega_1)$ **Net Demand** for good 1 and $(x_2 - \omega_2)$ **net demand** for good 2. The difference between what you want and what you started with.

x_1, x_2 **Gross Demands.**

Suppose $\omega_1 = 5$ and $\omega_2 = 5$ but I want to consume the bundle $(10, 0)$ (gross demand)

Net demand for good 1 is 5. The consumer is a **net buyer** of good 1.

Net demand for good 2 is -5. The consumer is a **net seller** of good 2.

$$p_1 (x_1 - \omega_1) + p_2 (x_2 - \omega_2) = 0$$

The cost of your net demands has to be zero. If a consumer is a net buyer of one good they have to be a net seller of the other.

1.4 Drawing the Budget Line and Changes to Price

Let's look at how the budget line change when we change prices. Suppose we have a consumer with $(\omega_1, \omega_2) = (5, 5)$. The budget:

$$p_1 x_1 + p_2 x_2 = 5p_1 + 5p_2$$

Notice that the consumer can always just consume their endowment. Thus, the endowment should be on the budget line regardless of the prices. This leads to the following: **When prices change the slope of the budget line changes: $-\frac{p_1}{p_2}$, but instead of pivoting through an endpoint, it pivots through the endowment.**

1.5 Price Changes and Net Buyers/Sellers

We have some strong predictions about what happens to consumer demand and utility when prices change:

- If a consumer is a **net seller of a good**, and the price of that good goes **up**, they **will remain a net seller of that good** and **will be strictly better off.**

- If a consumer is a **net buyer of a good**, and the price of that good goes **down**, they **will remain a net buyer of that good** and **will be strictly better off.**

1.6 Example Problem

Suppose we have an apple farmer with an endowment of $w_1 = 10$ apples and $w_2 = 0$ crusts. Their utility function is $u = min\left\{\frac{1}{2}x_1, x_2\right\}$. Initially the prices are $p_1 = 1$, $p_2 = 1$.

Budget equation under these prices and endowment:

$$1x_1 + 1x_2 = 1\,(10) + 1\,(0)$$

$$x_1 + x_2 = 10$$

To solve for demand:

$$\frac{1}{2}x_1 = x_2$$

$$x_1 + x_2 = 10$$

Solve them:

$$x_1 + \frac{1}{2}x_1 = 10$$

$$\frac{3}{2}x_1 = 10$$

$$x_1 = \frac{2}{3}10 = \frac{20}{3.0} = 6.66667$$

$$x_2 = \frac{1}{2}\left(\frac{20}{3}\right) = \frac{10}{3} = 3.33334$$

Is the consumer a net buyer or a net seller of apples?

Net seller of apples and a net buyer of crusts.

Suppose the price of apples increases to $p_1 = 2$. Will the consumer be a net buyer or a net seller of apples after the price change?

If a consumer is a net seller of a good and the price goes up, they will **remain a net seller and they will be strictly better off.**

$$\frac{1}{2}x_1 = x_2$$

$$2x_1 + x_2 = 20$$

Demand:

$$x_1 = 2\frac{20}{5}, x_2 = \frac{20}{5}$$

$$x_1 = 8, x_2 = 4$$

Remains a net seller of apples since the gross demand for apples (8) **is smaller than the endowment of apples** (10)**. And they are better off since their new utility is** 4 **and old utility was** $3\frac{1}{3}$**.**

2 Intertemporal Choice

How do consumers decide between borrow and saving money?

2.1 Bundles (Consumption Today, Consumption Tomorrow)

Bundles are going to in terms of a amount of money you choose use/consumer today (c_1) and consumption some time in the future (c_2).

Endowments are going to be in terms of income today (m_1), (m_2).

Bundle "consumed" is (c_1, c_2) and the stream of incomes (m_1, m_2).

2.2 Generating the Budget Constraint

A budget constraint needs prices. What takes the place of prices is the "interest rate". Interest rate r represents the extra money you get if you save or how much extra money you have to pay back if you borrow.

Let's suppose the consumer saves money today. If they save money today then $m_1 > c_1$ (receive more than spend). Thus, $m_1 - c_1 > 0$. $m_1 - c_1$ represents the amount saved:

How much can they spend tomorrow if I save $m_1 - c_1$ today?

If there is no interest. (Savings doesn't grow) they can consumer their income tomorrow (m_2) plus the amount saved $(m_1 - c_1)$

$$c_2 = (m_2) + (m_1 - c_1)$$

If there is interest (savings grows) they also get some extra back $r\,(m_1 - c_1)$:

$$c_2 = (m_2) + (m_1 - c_1) + r\,(m_1 - c_1)$$

This can be rewritten:

$$c_2 = (m_2) + (1 + r)\,(m_1 - c_1)$$

2.3 Example of Writing the Budget Equation

Suppose $m_2 = 10000$ and $m_1 - c_1 = 1000$ and $r = 0.1$ (10% interest rate).

$$c_2 = 10000 + (1 + 0.1)\,(1000) = 11100$$

$$10000 + 1100 = 11100$$

2.4 Writing the Full Budget Equation

Let's rearrange the equation we generated above:

$$c_2 = (m_2) + (1 + r)(m_1 - c_1)$$

$$c_2 = (m_2) + (1 + r) m_1 - (1 + r) c_1$$

$$(1 + r) c_1 + c_2 = (1 + r) m_1 + m_2$$

Now this looks like a budget equation we are used to, we can treat the price of good one (c_1) as $(1 + r)$ and the price of good two (c_2) as 1.

2.5 Example Problem

Suppose $m_1 = 200$, $m_2 = 600$, and $r = \frac{1}{2}$. Utility is: $u(c_1, c_2) = c_1 c_2$.

$$(1 + 0.5) c_1 + c_2 = (1 + 0.5)(200) + 600$$

$$(1 + 0.5) c_1 + c_2 = 300. + 600$$

Budget equation:

$$(1 + 0.5) c_1 + c_2 = 900.$$

Equal slope (tangncy)condition:

$$-\frac{c_2}{c_1} = -\frac{1.5}{1}$$

$$\{\{c_1 \to 300., c_2 \to 450.\}\}$$

Borrow 100 today. That will allow them to consumer 300. They have to pay back the 100 they borrow plus the interest which is $0.5(100) = 50$. Pay back 150 which allows them to consume $600 - 150 = 450$.