1 Intertemporal Choice

1.1 Two Forms of the Budget Equation

Future value of the budget equation:$(1+r)\,c_1 + c_2 = (1+r)\,m_1 + m_2$
Future value of income: $(1+r)\,m_1 + m_2$
Present value of the budget equation: $c_1 + \frac{1}{(1+r)}c_2 = m_1 + \frac{1}{(1+r)}m_2$
Present value of income: $m_1 + \frac{1}{(1+r)}m_2$

1.2 Another Example Problem

Suppose $m_1 = 200$, $m_2 = 600$, and $r = \frac{1}{2}$. Utility is: $u\,(c_1, c_2) = min\,\{c_1, c_2\}$.
Optimality requires $c_1 = c_2$ (no waste condition). Implies the consumer totally smooths their consumption.
A) Write down the consumers budget equation in terms of future value of income.

$$\left(1 + \frac{1}{2}\right) c_1 + c_2 = \left(1 + \frac{1}{2}\right) 200 + 600$$

What the future value of the consumers income?

$$\left(1 + \frac{1}{2}\right) (0) + c_2 = \left(1 + \frac{1}{2}\right) 200 + 600$$

$$c_2 = \left(1 + \frac{1}{2}\right) 200 + 600$$

$$900$$

What the present value of the consumers income?

$$c_1 + \frac{1}{\left(1 + \frac{1}{2}\right)} c_2 = 200 + \frac{1}{\left(1 + \frac{1}{2}\right)} 600$$

$$c_1 = 200 + \frac{1}{\left(1 + \frac{1}{2}\right)} 600$$

$$600$$

D) What the consumer's demand for c_1, c_2?

No Waste:

$$c_1 = c_2$$

Budget:

$$\left(1+\frac{1}{2}\right)c_1 + c_2 = \left(1+\frac{1}{2}\right)200 + 600$$

$$\left(1+\frac{1}{2}\right)c_1 + c_2 = 900$$

Plug the no waste into budget:

$$\left(1+\frac{1}{2}\right)c_1 + c_1 = 900$$

$$\frac{5}{2}c_1 = 900$$

$$c_1 = \frac{2}{5}900$$

$$c_1 = 360$$

By no waste:

$$c_2 = 360$$

$$(360, 360)$$

Are they a borrow or lender?

$$(m_1, m_2) = (200, 600)$$

Borrows 160**.**

Choosing to consume 360 but they get 600. This gives them $600 - 360 = 240$ to pay back their loan.

$$\left(1+\frac{1}{2}\right)160 = 240$$

What happens if the interest rate goes down to $\frac{1}{4}$? Is the consumer a borrow or lender?

They will remain a borrower and be strictly better off.

Check this at home.

1.3 Comparative Statics

If a consumer is **borrower** and the **interest rate goes down**, they remain a borrows and they are strictly better off.

If a consumer is **lender/saver** and the **interest rate goes up**, they remain a lender and they are strictly better off.

2 Market Demand

Until now our models have included just one consumer. In a market, in an economy we need to be able to handle more than one.

Turn individual demands into market demand that we can then use in models of the economy. The goal is to derive a downward sloping demand curve just like you saw in introductory economics.

2.1 Adding Demand Curves

People are denoted with numbers in subscripts. Goods are denoted with numbers in superscripts.

Individual demand for person i good 1: $x_i^1\left(p_1, p_2, m_i\right)$

Demand for person 1 good 2: x_1^2

Demand for person 5 good 1: x_5^1

Market demand is the sum of individual demands.

Market (aggregate) Demand for good 1:

Where n is the number of consumers:

$$X^1\left(p_1, p_2, m_1, ..., m_n\right) = \sum_{i=1}^{n} x_i^1\left(p_1, p_2, m_i\right)$$

If there are three consumers:

$$X^1\left(p_1, p_2, m_1, m_2, m_3\right) = x_1^1\left(p_1, p_2, m_1\right) + x_2^1\left(p_1, p_2, m_2\right) + x_3^1\left(p_1, p_2, m_3\right)$$

2.2 Example of Cobb-Douglas Demand

Suppose we have three people in the economy who each have cobb douglass preferences and their demands for good 1 and good 2 are:

Incomes of the consumers are $m_1 = 10, m_2 = 20, m_3 = 30$

$x_i^1 = \frac{\frac{1}{2}m_i}{p_1}$ and $x_i^2 = \frac{\frac{1}{2}m_i}{p_2}$.

Individual demands for good 1 are:

$$x_1^1 = \frac{\frac{1}{2}10}{p_1}, x_2^1 = \frac{\frac{1}{2}(20)}{p_1}, x_3^1 = \frac{\frac{1}{2}(30)}{p_1}$$

$$x_1^1 = \frac{5}{p_1}, x_2^1 = \frac{10}{p_1}, x_3^1 = \frac{15}{p_1}$$

Market demand for good 1 :

$$X^1 = \frac{5}{p_1} + \frac{10}{p_1} + \frac{15}{p_1}$$

$$X^1 = \frac{30}{p_1}$$

The is the exact same demand as if we had given any one of the consumer's all of the 60 of income in the economy.

Aggregate income:

$$M = m_1 + m_2 + m_3$$

$$M = 10 + 20 + 30 = 60$$

What would one of these consumer's demand if we gave them all the money in the economy:

$$x_i^1(p_1, p_2, M) = \frac{\frac{1}{2}(60)}{p_1} = \frac{30}{p_1}$$

2.3 Representative Consumer Condition

In the case above the market demand is the same as the demand from an individual consumer if we gave them the aggregate income in the economy.

When we can do this, we say the model has the **representative consumer property.**

Two conditions to have this:

1. Preferences have to be the same for everyone.
2. Their preferences have to be **homothetic.**

2.4 Homothetic Preferences

Preferences are **homothetic** if
$(x_1, x_2) \succsim (y_1, y_2)$ implies that $t(x_1, x_2) \succsim t(y_1, y_2)$ for all possible t.
Suppose I knew:

$$(2, 1) \succsim (1, 2)$$

If preferences are homothetic, then the following also have to be true:
Double the bundles:

$$2(2, 1) \succsim 2(1, 2)$$

$$(4, 2) \succsim (2, 4)$$

Cut the bundles in half

$$\frac{1}{2}(2, 1) \succsim \frac{1}{2}(1, 2)$$

$$\left(1, \frac{1}{2}\right) \succsim \left(\frac{1}{2}, 1\right)$$

With homothetic preferences all that matters is the proportions (ratio) of the goods but the not the scale (magnitude).

For any two bundles with the same proportions will have the same preferences.

Indifference curves that are parallel along a ray through the origin.

- Cobb Douglass
- Perfect Substitutes
- Perfect Complements

Quasi-linear is not the homothetic.

To check this, see if the MRS only depends on the ratio of goods. So that if you multiple both goods by some number, you get the same MRS.

Cobb douglass:

$$u(x_1, x_2) = x_1 x_2$$

$$MRS = -\frac{x_2}{x_1}$$

Multiply both by 2:

$$-\frac{2x_2}{2x_1} = -\frac{x_2}{x_1}$$

2.5 Representative consumer property fails for quasi-linear:

$u(x_1, x_2) = ln(x_1) + x_2$
Notice that the marginal rate of substitution does not depend just on the ratio.

$$MRS = -\frac{1}{x_1}$$

If we scale up consumption by 2, we get a different MRS:

$$-\frac{1}{2x_1}$$

These preferences are not homothetic.
Let's look at how this works out given actual demand:
$m_1 = 10, m_2 = 20, m_3 = 30$
$p_1 = 1, p_2 = 1$

$$MRS = -\frac{p_1}{p_2}$$

$$x_1 + x_2 = m_i$$

$$-\frac{\frac{1}{x_1}}{1} = -1$$

$$x_1 = 1$$

Plug this back into the budget equation:

$$x_2 = m_i - 1$$

$$(1, m_i - 1)$$

Market demands:

$$x_1^1 = 1, x_2^1 = 1, x_3^1 = 1$$

$$x_1^2 = 9, x_2^2 = 19, x_3^2 = 29$$

$$X^1 = 3$$

$$X^2 = 57$$

Representative consumer property fails. If we gave all 60 to some consumer 1:

$$x_1^1\left(1,1,M\right) = x_1^1\left(1,1,60\right)$$

$$x_1^1\left(p_1 = 1, p_2 = 1, 60\right) = 1$$

$$x_1^2\left(p_1 = 1, p_2 = 1, 60\right) = 59$$