## 0.1 Elasticities

A way of measure a rate of change that is "unit free".

Normally when measure how something changes in response to a change in something else, the magnitude depends on how we measure those things.

How does demand change when we change price?

Price of apples $p = 1$ and market demand $X = 1000$.

I price increases to $p = 2$ market demand drops to $X = 500$.

The change is a decrease of 500 for a one dollar increase in price.

Let's measure demand in dozens.

$$p = 1, X = 83.3333$$

$$p = 2, X = 41.6667$$

One dollar increase in price leads to a 41.6666 decrease in demand for dozens of apples.

**No matter how we measure this, there is a 100% increase in price and a 50% decrease in demand.** This is a rate of change that doesn't depend on units.

This also works for derivatives:

$$\frac{\partial x}{\partial p}$$

The derivative is a rate of change, it is also dependent on how we measure things.

An elasticity is like a derivative except it doesn't depend on units of measure. Instead it is in terms of percentages.

Change in demand relative to the change in price:

$$\frac{\Delta x}{\Delta p}$$

$$\frac{(1000 - 500)}{(2 - 1)} = \frac{500}{1}$$

$$\frac{\frac{1000-500}{1000}}{\frac{2-1}{1}} = \frac{0.5}{1} = 0.5$$

Or 50%

To move from a change to a percent change:

$$\frac{\Delta X}{\Delta p} \rightarrow \frac{\frac{\Delta X}{X}}{\frac{\Delta p}{p}}$$

$$\frac{\frac{\Delta X}{X}}{\frac{\Delta p}{p}} \rightarrow \frac{\frac{\partial X}{X}}{\frac{\partial p}{p}}$$

This is now a derivative measured in percentage terms.

Instead of asking (derivative) how much does on thing change when we change another thing by a "little bit". We ask what is the percent change in one thing when the change the other thing by some very small percentage.

**Elasticity: What is the percent change in one thing when the other thing increases by 1%?**

Price elasticity:

What is the percent change in demand when the price of that good goes up by 1%?

$$\varepsilon_{1,1} = \frac{\frac{\partial x_1}{x_1}}{\frac{\partial p_1}{p_1}} = \frac{\partial x_1}{\partial p_1} \frac{p_1}{x_1}$$

Cross Price elasticity:

What is the percent change in demand when the price of **the other good** goes up by 1%?

$$\varepsilon_{1,2} = \frac{\frac{\partial x_1}{x_1}}{\frac{\partial p_2}{p_2}} = \frac{\partial x_1}{\partial p_2} \frac{p_2}{x_2}$$

Income elasticity:

What is the percent change in demand when income increases by 1%?

$$\eta_1 = \frac{\frac{\partial x_1}{x_1}}{\frac{\partial m}{m}} = \frac{\partial x_1}{\partial m} \frac{m}{x_1}$$

## 0.2 Cobb-Douglas Example

Suppose utility is $u = x_1 x_2$. Demand for good 1 is: $x_1 = \frac{\frac{1}{2}m}{p_1}$.

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

Price elasticity:

$$\varepsilon_{1,1} = \frac{\partial x_1}{\partial p_1}\frac{p}{x_1}$$

$$= \frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial p_1}\frac{p_1}{\frac{\frac{1}{2}m}{p_1}}$$

Take the derivative:

$$\frac{\partial\left(\frac{1}{2}mp_1^{-1}\right)}{\partial p_1}\frac{p_1}{\frac{\frac{1}{2}m}{p_1}} = \left(-1\frac{1}{2}mp_1^{-2}\right)\frac{p_1}{\frac{\frac{1}{2}m}{p_1}}$$

$$= \frac{-\frac{1}{2}m}{p_1^2}\frac{p_1}{\frac{\frac{1}{2}m}{p_1}}$$

$$= \frac{-\frac{1}{2}m}{p_1^2}\frac{p_1^2}{\frac{1}{2}m}$$

$$= \frac{-\frac{1}{2}m}{p_1^2}\frac{p_1^2}{\frac{1}{2}m} = -1$$

Price elasticity of demand for a good is negative one.
*"When price goes up by 1% demand goes down by 1%"*
Income elasticity:
Intuition tells us the answer should be 1.

$$\eta_1 = \frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial m}\frac{m}{\frac{\frac{1}{2}m}{p_1}}$$

$$= \frac{\frac{1}{2}}{p_1}\frac{m}{\frac{\frac{1}{2}m}{p_1}}$$

$$= \left(\frac{\frac{1}{2}}{p_1}\right)\frac{m}{m\left(\frac{\frac{1}{2}}{p_1}\right)} = 1$$

Cross price elasticity:

$$\varepsilon_{1,2} = \frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial p_2}\frac{p_2}{\frac{\frac{1}{2}m}{p_1}} = 0\frac{p_2}{\frac{\frac{1}{2}m}{p_1}} = 0$$

## 0.3 An Aside on Cobb-Douglass

$u = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$

$$MRS = -\frac{\frac{\partial\left(x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}\right)}{\partial x_1}}{\frac{\partial\left(x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}\right)}{\partial x_2}} = -\frac{1}{2}\frac{x_2}{x_1}$$

This isn't the demand:

$$\left(\frac{\frac{1}{2}m}{p_1}, \frac{\frac{1}{2}m}{p_2}\right)$$

This is:

$$\left(\frac{\frac{\frac{1}{3}}{\frac{1}{3}+\frac{2}{3}}m}{p_1}, \frac{\frac{\frac{2}{3}}{\frac{1}{3}+\frac{2}{3}}m}{p_2}\right) = \left(\frac{\frac{1}{3}m}{p_1}, \frac{\frac{2}{3}m}{p_2}\right)$$

If we have: $x_1^\alpha x_2^\beta$

$$\left(\frac{\frac{\alpha}{\alpha+\beta}m}{p_1}, \frac{\frac{\beta}{\alpha+\beta}m}{p_2}\right)$$

$x_1^2 x_1^1$

$$\left(\frac{\frac{2}{3}m}{p_1}, \frac{\frac{1}{3}m}{p_2}\right)$$

$\frac{\alpha}{\alpha+\beta} + \frac{\beta}{\alpha+\beta} = 1$

## 0.4 Classifications of Price Elasticity

Imagine we had a good for which $\varepsilon_{1,1} = -50$.

When price goes up by 1% demand drops by 50%.

**Elastic Goods**

When price elasticity is less than $-1$ demand reacts strongly to changes in price. We say that they are "Elastic".

Goods which have strong substitutes tend to be elastic. Goods that are not necessities.

**Inelastic Goods**

Price elasticity $\varepsilon_{1,1} > -1$.

Demand has a very weak reaction to a change in price.

$\varepsilon_{1,1} = -0.01$

Demand goes down by 0.01% when price goes up by 1%.

Gasoline. Medications. Things you are addicted to. Staple foods.

$\varepsilon_{1,1} = -1$ “Unit elastic”.