1 Fall 2023 Midterm Question 5.

$x = \frac{m+100}{p}$.

1. What is each consumer's income elasticity?

Income elasticity is $\frac{\partial x}{\partial m}\frac{m}{x}$

$$\frac{\partial\left(\frac{m+100}{p}\right)}{\partial m}\frac{m}{\frac{m+100}{p}}$$

$$\frac{\partial\left(\frac{m+100}{p}\right)}{\partial m}\frac{m}{\frac{m+100}{p}}$$

$$\frac{\partial\left(\frac{m}{p}+\frac{100}{p}\right)}{\partial m}\frac{m}{\frac{m+100}{p}}$$

$$\left(\frac{\partial\frac{m}{p}}{\partial m}+\frac{\partial\left(\frac{100}{p}\right)}{\partial m}\right)\frac{m}{\frac{m+100}{p}}$$

$$\frac{1}{p}\frac{m}{\frac{m+100}{p}}$$

$$\frac{1}{p}m\frac{p}{m+100}$$

$$=\frac{m}{m+100}$$

Since $\frac{m}{m+100} < 1$. If income goes up by 1% demand will go up by less than 1% regardless of income.

An Aside. Suppose we had $\frac{\partial\left(\frac{\frac{1}{2}m}{p}\right)}{\partial m}\frac{m}{\frac{\frac{1}{2}m}{p}}$

$$\frac{1}{2p}\frac{m}{\frac{\frac{1}{2}m}{p}}$$

$$\frac{2p}{2p}\frac{m}{m}=1$$

B. Three consumers have $x = \frac{m+100}{p}$. $X = \frac{m+100}{p} + \frac{m+100}{p} + \frac{m+100}{p}$

$$X = 3\frac{m+100}{p}$$

C. Plot Inverse demand when $m = 100$.

$$X = 3\frac{m+100}{p}$$

$$p = 3\frac{m+100}{X}$$

$$3\frac{100+100}{X}$$

$$p = \frac{3\,(200)}{X} = \frac{600}{X}$$

An Aside. At $m = 100$, what price would result a market demand of 100?

$$p = \frac{600}{X}$$

$$p = \frac{600}{100} = 6$$

At a price of 6 **consumers will buy a total of** 100 **units.**

D. If we were to give one consumer all of the income in the market, would they the same amount as the actual market demand?

Market demand: $X = 3\frac{m+100}{p} = \frac{3m+300}{p}$

If we gave one consumer all the income in the market $M = 3m$. They would consume:

$$x = \frac{(3m)+100}{p} = \frac{3m+100}{p}$$

$$\frac{3m+100}{p} \neq \frac{3m+300}{p}$$

Can't use representative consumer property here.

2 Review

2.1 Budget Line

2.1.1 Income

$$p_1x_1 + p_2x_2 = (m)$$

2.1.2 Endowments

$$p_1 x_1 + p_2 x_2 = (p_1 \omega_1 + p_2 \omega_2)$$

2.1.3 Intertemporal Choice

$$(1+r)\, c_1 + c_2 = (1+r)\, m_1 + m_2$$

$$c_1 + \frac{1}{1+r} c_2 = m_1 + \frac{1}{1+r} m_2$$

2.1.4 Taxes

$$(p_1 + t)\, x_1 + p_2 x_2 = m$$

2.1.5 Description of Homotheticity.

If preferences are homothetic, their willingness to trade off between two goods doesn't depend on how much of the goods they have, it only depends on the proportion or ratio of the two goods.

2.1.6 Cobb Douglass is Homothetic

Check the if you scale up x_1 and x_2 by some number (here 2) you get the same MRS.

$$-\frac{x_2}{x_1}$$

$$-\frac{2x_2}{2x_1} = -\frac{x_2}{x_1}$$

3 Monotonic Preferences

$$u(x_1, x_2) = x_1 x_2$$

Not monotonic

$$u(x_1, x_2) = x_1 - x_2$$

3.1 Convex Preferences

3.2 MRS

$$u\left(x_1, x_2\right)$$

$$MRS = -\frac{\frac{\partial(u(x_1,x_2))}{\partial x_1}}{\frac{\partial(u(x_1,x_2))}{\partial x_2}}$$

How much x_2 would I be willing to give up to get one more unit of x_1. For utility $x_1^2 x_2^3$:

$$-\frac{\frac{\partial\left(x_1^2 x_2^3\right)}{\partial x_1}}{\frac{\partial\left(x_1^2 x_2^3\right)}{\partial x_2}} = -\frac{2x_1 x_2^3}{3x_1^2 x_2^2} = -\frac{2}{3}\frac{x_2}{x_1}$$