1 Demand

Budget Set

$p_1x_1 + p_2x_2 \leq m$

If preferences are monotonic (more is better), consume on the budget line:

$p_1x_1 + p_2x_2 = m$

$u(x_1, x_2) = x_1x_2$

1.1 Interpreting the MRS=Price Ratio Condition

$$MRS = -\frac{x_2}{x_1}$$

Optimum occurs where the MRS is equal to the price ratio (slope of the budget line).

$$MRS = -\frac{p_1}{p_2}$$

Another interpretation.

$$-\frac{MU_1}{MU_2} = -\frac{p_1}{p_2}$$

$$\frac{MU_1}{p_1} = \frac{MU_2}{p_2}$$

This is the same expression but rearranged.

Notice that $\frac{MU_i}{p_i}$ is a measure of how much utility you get buy spending one more dollar on good i.

$$MU_1 = 1, p_1 = 2$$

$$\frac{MU_1}{p_1} = \frac{1}{2}$$

$$MU_1 = 2, p_1 = 1$$

$$\frac{MU_1}{p_1} = 2$$

I think of $\frac{MU_i}{p_i}$ as the "bang for you buck" for each"

$$\frac{MU_1}{p_1} = \frac{MU_2}{p_2}$$

Suppose otherwise:

$$\frac{MU_1}{p_1} > \frac{MU_2}{p_2}$$

Spend more on good 1.

$$\frac{MU_1}{p_1} < \frac{MU_2}{p_2}$$

Spend more on good 2.

1.2 Example Cobb Douglass Preferences

$2x_1 + 1x_2 = 20$
$u(x_1, x_2) = x_1 x_2$

$$MRS = -\frac{\frac{\partial(x_1 x_2)}{\partial x_1}}{\frac{\partial(x_1 x_2)}{\partial x_2}} = -\frac{x_2}{x_1}$$

At the optimum, **equal slope condition (tangency condition).**

$$MRS = -\frac{p_1}{p_2}$$

$$-\frac{x_2}{x_1} = -\frac{2}{1}$$

Budget Line

$$2x_1 + x_2 = 20$$

Solve the system of equations:

$$-\frac{x_2}{x_1} = -\frac{2}{1}$$

$$x_2 = 2x_1$$

Plug this into the budget equation:

$$2x_1 + x_2 = 20$$

$$2x_1 + 2x_1 = 20$$

$$4x_1 = 20$$

$$x_1 = 5$$

Plug this back into $x_2 = 2x_1$ to get x_2:

$$x_2 = 2\,(5)$$

$$x_2 = 10$$

Optimal bundle $(5, 10)$.

1.2.1 Marshallian Demand

When we solve for demand "generically" for unspecified p_1, p_2, m we get the **marshallian demand.** Tells us the optimal amount of a good for any price and income we might encounter.

$$p_1x_1 + p_2x_2 = m$$

$$u\,(x_1, x_2) = x_1x_2$$

Two conditions for optimality (equal slope and budget):

$$-\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

$$p_1x_1 + p_2x_2 = m$$

Solve this system of equations:

$$-\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

$$p_2x_2 = p_1x_1$$

This says "spend the same on both goods" and thus implies I will **spend half of my money on good one and half on good two**.

$$p_1x_1 + p_2x_2 = m$$

Plug in the condition above:

$$p_1x_1 + p_1x_1 = m$$

$$2p_1x_1 = m$$

$$p_1x_1 = \frac{m}{2}$$

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

Plug this back into the equal slope condition to get good 2:

$$p_1x_1 = p_2x_2$$

$$p_1\left(\frac{\frac{1}{2}m}{p_1}\right) = p_2x_2$$

$$\frac{1}{2}m = p_2x_2$$

$$x_2 = \frac{\frac{1}{2}m}{p_2}$$

Marshallian Demands for utility function $u\left(x_1, x_2\right) = x_1x_2$

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

$$x_2 = \frac{\frac{1}{2}m}{p_2}$$

Let's check that with $p_1 = 2, p_2 = 1, m = 20$ we get the demand we found before:

$$x_1 = \frac{\frac{1}{2}20}{2}, x_2 = \frac{\frac{1}{2}20}{1}$$

$$x_1 = 5, x_2 = 10$$

1.2.2 Changes in Income

We look how demand changes when income changes. There are two possibilities:

Normal: Demand goes up when income goes up.

Examples: Fresh Produce, Kombucha, Chanel Handbags, Roelx Watches

Inferior: Demand goes down when income goes up.

Examples: Ramen Noodles, Cheap Beer, Fast/Processed Food

From our Cobb Douglass example:

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

$$\frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial m} = \frac{1}{2p_1} > 0$$

Normal Good.

A good can be "always normal" but it can't be "always inferior".

This is because in order for demand to decrease, it will have had to increase at some point, so it **has** to start out normal.

1.3 Engel Curve

This is a plot of demand for a good against income. Our natural reaction would be to put m (income) on the horizontal axis and x_i demand on the vertical axis. **This is not what we do**.

Take demand for a cobb douglass consumer above when $p_1 = 2$

$$x_1 = \frac{\frac{1}{2}m}{2} = \frac{1}{4}m$$

$$x_1 = \frac{1}{4}m$$

For the engle curve we plot m aginst x_1 so let's isolate m :

$$m = 4x_1$$

1.3.1 Changes in "Own" Price

How does demand for a good change when we change the price of **that good**.

Ordinary: When the price of a good goes up demand goes down.

Examples: Basically everything.

Giffen: When the price of a good goes up demand goes up.

If a good is "so inferior" that when the price of that good goes up and my effective income goes down, I end up buying even more than I did before.

Example: Gruel