

0.1 Recap

Marshaillian Demand:

$$x_1(p_1, p_2, m)$$

Income (m)

If income goes up and demand goes up: **normal**

If income goes up and demand goes down: **inferior**

Engle Curve: Plot of income (vertical axis) against demand (horizontal)

Own Price (p_1)

If price goes up and demand goes down: **ordinary**

If price goes up and demand goes up: **giffen**

Inverse Demand Curve: Plot of price (vertical axis) against demand (horizontal)

0.2 Example of Inverse Demand

$$u(x_1, x_2) = x_1x_2.$$

Marshallian Demand for good 1: $x_1^* = \frac{\frac{1}{2}m}{p_1}$

Plot the engle curve for $p_1 = 1$

$$x_1^* = \frac{1}{2}m$$

$$m = 2x_1$$

Plot the inverse demand curve when $m = 10$

$$x_1^* = \frac{\frac{1}{2}(10)}{p_1}$$

$$x_1^* = \frac{5}{p_1}$$

$$p_1 = \frac{5}{x_1}$$

0.3 Changes in “Other” Price

How does demand for x_1 change when we change p_2 ?

Substitutes: If demand for x_1 goes up when p_2 goes up.

Complements: If demand for x_1 goes down when p_2 goes up.

Neither: Demand for x_1 does not depend on p_2 .

0.3.1 Example of Perfect Complements

$$u = \min \{x_1, x_2\}$$

$$x_1 = x_2$$

$$p_1x_1 + p_2x_2 = m$$

Plug in no-waste into the budget equation:

$$p_1x_1 + p_2x_1 = m$$

$$(p_1 + p_2)x_1 = m$$

$$x_1 = \frac{m}{p_1 + p_2}$$

$$x_2 = \frac{m}{p_1 + p_2}$$

What happens to x_1 when p_2 increases? It goes down because p_2 only appears in the denominator.

$$\frac{\partial \left(\frac{m}{p_1 + p_2} \right)}{\partial p_2} = -\frac{m}{(p_1 + p_2)^2}$$

0.3.2 Example of Perfect Substitutes

$u = x_1 + x_2$ suppose $m = 10$, $p_1 = 1$ and $p_2 = 2$.

$$x_1 = 10, x_2 = 0$$

If p_1 goes up to $p_1 = 3$?

$$x_1 = 0, x_2 = 5$$

p_1 went up and x_2 went up so they are **substitutes**.

0.3.3 Example of Cobb-Douglas

Suppose $u = x_1x_2$.

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

$$x_2 = \frac{\frac{1}{2}m}{p_2}$$

Neither complements nor substitutes.

1 Slutsky Decomposition

When price changes for good, there are two reasons why demand for that good can change.

Substitution Effect. Demand for the good goes down because I substitute into buying other things.

Income Effect. When the price changes, and I continue to buy that good, I can buy of less of it because my income is now effectively lower.

1.1 Slutsky Decomposition

The Slutsky decomposition is a “formal” thought experiment.

If the total change in demand is the sum of the income effect and substitution effect then if we can hold one of these constant, we can directly measure the other.

The Slutsky decomposition holds the income effect constant and thus allows us to directly measure the substitution effect.

What we do is imagine giving the consumer extra income so that they can buy the bundle they were buying before the price change. Then we ask what would buy at the new prices but with the extra income. Any change to demand **cannot be due to the income effect**. It must be substitution.

We **isolate the substitution effect** by giving the consumer extra income so that they can buy what they were buying before at the new price.

1.2 Example Problem

Suppose $u = x_1x_2$.

Demand is $x_1^* = \frac{\frac{1}{2}m}{p_1}, x_2^* = \frac{\frac{1}{2}m}{p_2}$.

Suppose $p_1 = 1, p_2 = 2, m = 20$.

What is the bundle the consumer buys before the price change?

$$(10, 5)$$

Now suppose p_1 changes to $p_1 = 2$

$$(5, 5)$$

The total change in demand (total effect) is 5.

To find the substitution effect, we imagine giving the consumer **enough income to buy the old bundle** at the new prices to wipe out the effect price has on relative income.

The income they need to buy the old bundle at the new prices. The bundle $(10, 5)$ now costs 30 when $p_1 = 2$ and $p_2 = 2$. Let's imagine giving the consumer $m = 30$ but at the prices.

$$\left(\frac{\frac{1}{2}30}{2}, \frac{\frac{1}{2}30}{2}\right)$$

$$\left(\frac{15}{2}, \frac{15}{2}\right)$$

$$(7.5, 7.5)$$

They were buying 10 before the price change, and 7.5 after the price change but with extra income. This 2.5 unit decrease **cannot be due to income effect. Must be substitution.**

Of the 5 unit change in demand 2.5 is due to substitution. The remainder (2.5) is due to the income effect.