1 Profit Maximization / Cost Minimization

1.1 Consumer Problem / Producer Problem

Consumers try to find the best bundle according to their preferences within their budget set, or *maximize utility subject to their budget.*

Firms maximize profits.

$$profit = revenue - cost$$

1.2 Formalizing the Profit Function

Firms choose inputs x_1, x_2 to produce output y through the production function:

$$y = f\left(x_1, x_2\right)$$

Revenue is price times quantity. Price is *potentially* a function of output y and is given by $p\left(y\right)$

$$rev = p\left(y\right) y$$

Since $y = f\left(x_1, x_2\right)$:

$$rev\left(x_1, x_2\right) = p\left(f\left(x_1, x_2\right)\right) f\left(x_1, x_1\right)$$

Cost is a function of the inputs they choose and the price of those inputs w_1, w_2.

$$c\left(x_1, x_2\right) = w_1 x_1 + w_2 x_2$$

1.3 Profit Function

Now we can write the profit function in terms of x_1 and x_2

$$\pi\left(x_1, x_2\right) = p\left(f\left(x_1, x_2\right)\right) f\left(x_1, x_2\right) - \left(w_1 x_1 + w_2 x_2\right)$$

1.4 Price Taking

The price function $p\left(y\right)$ can depend on the market structure. Is the firm a monopoly? Is there competition? Does government regulate price?

The simplest assumption we can make is that price does not depend on output. That is, **price is fixed**. *Price-taking assumption*, perfect-competition assumption.

This is a mathematically simplifying assumption only appropriate when the firm is a very small part of the market.

$p\left(y\right) = p$ **the price of output is fixed.**

1.5 Profit Under Price-Taking

$$\pi\left(x_1, x_2\right) = pf\left(x_1, x_2\right) - \left(w_1 x_1 + w_2 x_2\right)$$

1.6 Short vs Long Run

We say that a firm is operating in the **short run** if *some input is **fixed***.
We say that a firm is operating in the **long run** if *all of the inputs are variable.*

1.7 Example Profit Maximization- Short Run

Suppose $f\left(x_1, x_2\right) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$. Assume that x_2 is fixed at $\bar{x_2} = 1$. The price of output is $p = 4$ and the price on inputs are $w_1 = 1$ and $w_2 = 1$.
Short run production function: $f\left(x_1, 1\right) = x_1^{\frac{1}{2}}\left(1^{\frac{1}{2}}\right) = x_1^{\frac{1}{2}}$
Short run profit function:

$$\pi\left(x_1, 1\right) = 4\left(x_1^{\frac{1}{2}}\right) - \left(1x_1 + 1\left(1\right)\right)$$

$$\pi\left(x_1, 1\right) = 4x_1^{\frac{1}{2}} - x_1 - 1$$

To maximize profit, look for where marginal profit (derivative of profit with respect to the input) is zero. Where the slope is zero:

$$\frac{\partial\left(4x_1^{\frac{1}{2}} - x_1 - 1\right)}{\partial x_1} = 0$$

$$2x_1^{-\frac{1}{2}} - 1 = 0$$

$$x_1^{-\frac{1}{2}} = \frac{1}{2}$$

$$\frac{1}{x_1^{\frac{1}{2}}} = \frac{1}{2}$$

$$x_1^{\frac{1}{2}} = 2$$

$$x_1 = 4$$

The profit maximizing use of x_1 is $x_1 = 4$. This earns the firm:

$$\pi\left(4, 1\right) = 4\left(4^{\frac{1}{2}}\right) - 4 - 1 = 3$$

1.8 Profit Maximization- Long Run

$$\pi\left(x_1,x_2\right)=4\left(x_1^{\frac{1}{2}}x_2^{\frac{1}{2}}\right)-\left(x_1+x_2\right)$$

A two function is maximized where all of the partial derivatives are zero:

$$\frac{\partial\left(4\left(x_1^{\frac{1}{2}}x_2^{\frac{1}{2}}\right)-\left(x_1+x_2\right)\right)}{\partial x_1}=0$$

$$\frac{2\sqrt{x_2}}{\sqrt{x_1}}-1=0$$

$$\frac{\partial\left(4\left(x_1^{\frac{1}{2}}x_2^{\frac{1}{2}}\right)-\left(x_1+x_2\right)\right)}{\partial x_2}=0$$

$$\frac{2\sqrt{x_1}}{\sqrt{x_2}}-1=0$$

We need to solve $\frac{2\sqrt{x_2}}{\sqrt{x_1}}-1=0$ and $\frac{2\sqrt{x_1}}{\sqrt{x_2}}-1=0$ simultaneously. Mathematica says the answer is:

$$\{\}$$

... what happened?!?! It turns out there is no profit maximizing level of inputs. To see this more clearly, let's take the **better** approach to maximizing profit. *Cost minimization.*

1.9 Profit Maximization Requires Cost Minimization

If a firm is maximizing profit, it must be producing a certain amount of output.

If that amount of output isn't produced in the cheapest way, the firm could lower is costs while keeping revenue the same. That would increase profit.

Profit maximization requires cost minimization.

1.10 Cost Minimization

Our goal is two write the profit function **only in terms of output.**

1. Calculate the cheapest way to produce any level of output. Create the cost function $c(y)$ that will tell of the cost of producing output y in the cheapest way.

2. Construct a profit function that depends only on output:

$$\pi(y)=py-c(y)$$

1.11 Minimizing Cost for our Example:

f$(x_1, x_2) =$$x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$.
To minimize cost, look for a bundle of inputs where the slope of the isoquant is the same as the slope of the iso-cost:

$$-\frac{x_2}{x_1} = -\frac{w_1}{w_2}$$

$$x_1 w_1 = x_2 w_2$$

$$x_1 = \frac{x_2 w_2}{w_1}$$

In the previous example $w_1 = 1, w_2 = 1$.

$$x_1 = x_2$$

Production constraint:

$$x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = y$$

$$x_2^{\frac{1}{2}} x_2^{\frac{1}{2}} = y$$

$$x_2 = y$$

$$x_1 = y$$

1.12 Cost Function

To produce output y in the cheapest way use bundle (y, y) "conditional factor demands"

$$c(y) = 1y + 1y$$

$$c(y) = 2y$$

1.13 Optimal y

Profit function in terms of y:

$$\pi(y) = 4y - 2y = 2y$$

There is no profit maximizing level of output.