1 Monopoly

Under the price-taking assumption, this is the profit function for a firm (p is a fixed unit-price for output).

$$\pi(q) = pq - c(q)$$

1.1 Monopolies and the Price-Taking Assumption

Let's relax the price taking assumption, we will assume there is just one firm that produces the good.

Suppose demand is: $Q(p) = \frac{\frac{1}{2}(200)}{p}$ and there is one firm in a market.
The inverse demand function is:

$$q = \frac{100}{p}$$

$$p(q) = \frac{100}{q}$$

This tells us the most consumers are willing to pay for quantity q of this good. The most this firm can charge to sell 100 units is 1.

$$p(100) = 1$$

Suppose they increase their quantity to 200.

$$p(200) = \frac{100}{200} = 0.50$$

1.2 The Monopolist's Profit Function

We go from perfect competition

$$\pi(q) = pq - c(q)$$

Monopoly:

$$\pi(q) = p(q)q - c(q)$$

When we maximize this:

$$\frac{\partial(p(q)q - c(q))}{\partial q} = \frac{\partial p(q)}{\partial q}q + p(q) - \frac{\partial c(q)}{\partial q}$$

$$\frac{\partial p(q)}{\partial q}q + p(q) - \frac{\partial c(q)}{\partial q} = 0$$

1.3 Example of Maximizing Profit

Suppose demand is $q_d(p) = 100 - p$. Cost is $c(q) = 10q$.

$$q_d(p) = 100 - p$$

$$q = 100 - p$$

The inverse demand:

$$p(q) = 100 - q$$

The profit function:

$$\pi(q) = (100 - q)\, q - 10q$$

$$100q - q^2 - 10q$$

$$= 90q - q^2$$

Find the maximum by looking for a q where marginal profit is zero:

$$\frac{\partial\left(90q - q^2\right)}{\partial q} = 90 - 2q = 0$$

$$90 = 2q$$

$$q = 45$$

$$\begin{pmatrix} 20 & 1400 \\ 30 & 1800 \\ 40 & 2000 \\ 45 & 2025 \\ 50 & 2000 \\ 55 & 1925 \end{pmatrix}$$

Plug the profit maximizing quantity into the inverse demand to get price:

$$p(45) = 100 - 45 = 55$$

$$q = 45, p = 55$$

$$\pi(45) = 2025$$

1.4 Previous Example but Perfect Competition

$$pq - 10q$$

$$(p - 10)\, q$$

Suppose $p > 10$, then all firms want to produce an infinite amount.

If $p < 10$ then all firms want to produce 0.

Both of these are unsatisfactory solutions to the problem. We are left to conclude the price in this market must end up being 10.

$$p = 10$$

$$mc = 10$$

Relative to perfect competition, there is a sharp markup over marginal cost in the last example. For the monopoly:

$$\frac{p}{mc} = \frac{55}{10} = 5.5$$

1.5 What does a monopoly do?

What we want to do is write the markup term in terms of elasticity. In the previous example the markup term is 5.5, we want to see that this is driven by elasticity of demand.

$$\frac{\partial\left(p\left(q\right)q - c\left(q\right)\right)}{\partial q} = 0$$

$$\frac{\partial p}{\partial q} q + p - mc = 0$$

Elasticity of demand: $\varepsilon = \frac{\partial q}{\partial p}\frac{p}{q}$. $\frac{1}{\varepsilon} = \frac{\partial p}{\partial q}\frac{q}{p}$. If we divide both sides by p:

$$\frac{\partial p}{\partial q}\frac{q}{p} + 1 - \frac{mc}{p} = 0$$

$$\frac{1}{\varepsilon} + 1 = \frac{mc}{p}$$

$$\frac{1+\varepsilon}{\varepsilon} = \frac{mc}{p}$$

$$\frac{p}{mc} = \frac{\varepsilon}{1+\varepsilon}$$

Suppose $\varepsilon = -2$

$$\frac{p}{mc} = \frac{-2}{1-2} = 2$$

In the previous example we had:

$$\frac{p}{mc} = 5.5$$

$$5.5 = \frac{\varepsilon}{1+\varepsilon}$$

$$\varepsilon = -1.22222$$

Notice of these examples involve the firm operating where demand is elastic. This is because a monopolist could **never** be maximizing profit where demand is inelastic.

If demand is inelastic, then to get quantity to decrease by 1% I have to raise price by more than 1%. If I decrease quantity by 1%, I can increase price by more than 1%.

If price goes up by more than 1% and demand goes down by 1%, the revenue has to go up.

If demand is inelastic, the firm can always get more revenue by decreasing quantity.

If I decrease quantity, cost also goes down.

Thus, revenue goes up, cost goes down and profit must increase if demand is inelastic. (This also works for unit-elastic demand).

Monopolists always operate in the "elastic portion" of the demand curve.

1.6 Example with constant elasticity of demand -1.

Cost is $c(q) = q$

$$q = \frac{100}{p}$$

Elasticity of demand is -1. $\frac{\partial\left(\frac{100}{p}\right)}{\partial p}\frac{p}{\frac{100}{p}} = -1$

Inverse demand:

$$p(q) = \frac{100}{q}$$

$$\pi(q) = q\frac{100}{q} - q$$

$$\pi(q) = 100 - q$$

Technically this profit function is “maximized” at $q = 0$

1.6.1 Checking Elastic Portion for Previous Example

1.7 In Action

Suppose a firm is charging $p = 100$ and consumers have elasticity of demand -3. What is the firms marginal cost?

$$\frac{p}{mc} = \frac{\varepsilon}{1+\varepsilon}$$

$$\frac{100}{mc} = \frac{-3}{1-3}$$

$$\frac{100}{mc} = \frac{-3}{-2}$$

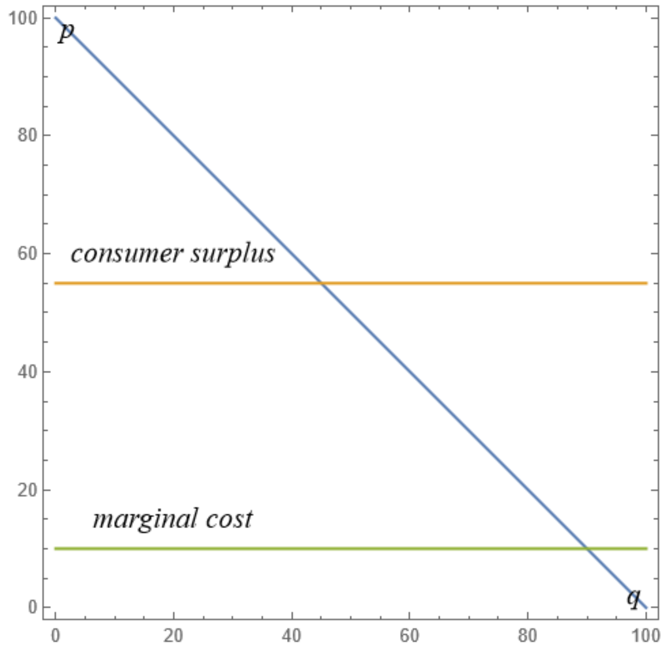
$$\frac{100}{1.5} = mc$$

$$66.6667 = mc$$

1.8 Consumer Surplus Under Monopoly

As we saw above, suppose a firm had constant marginal cost of $c(y) = 10y$ and demand is $q = 100 - p$. Optimal quantity was $q = 45$ and price is $p = 55$.

Inverse demand is $p(q) = 100 - q$



100
80
60
40
20
0
0
20
40
60
80
100
p
q
consumer surplus
marginal cost