1 Technology

Just like consumer's have preferences, firm's have "technology". Technology is a description of how inputs turn into outputs.

Suppose we have a baker with the following technology: they turn 1 **crust** and 2 **apples** into 1 **pie.** We can represent this with a "vector" like this:

(crusts,apples,pies)

Make one pie from one crust and two apples

$$(-1, -2, 1)$$

Make two pies from two crust and four apples

$$(-2, -4, 2)$$

This way of expressing a technology is very flexible, but we won't have to worry so much about dealing with vectors like this. We can represent a technology with a **production function.**

1.1 Production Functions

Production function takes in amounts of two inputs and returns an amount of output.

$f(x_1, x_2) = y$

A production function works just like a utility function. The practical difference is that the "number" that represents the output is inherently meaningful (cardinal).

1.2 Isoquants

We visualized prefrences for consumers through the indifference curves. We visualize production function through the isoquants.

Isoquants: Bundles of inputs that produce the same output.

Here are some bundles that all produce *one pie.* They are on the same isoquant. They are on the isoquant for one pie.

$$(1, 2), (1, 3), (2, 2)$$

Here are bundles on another isoquant:

$$(2, 4), (2, 5), (3, 4)$$

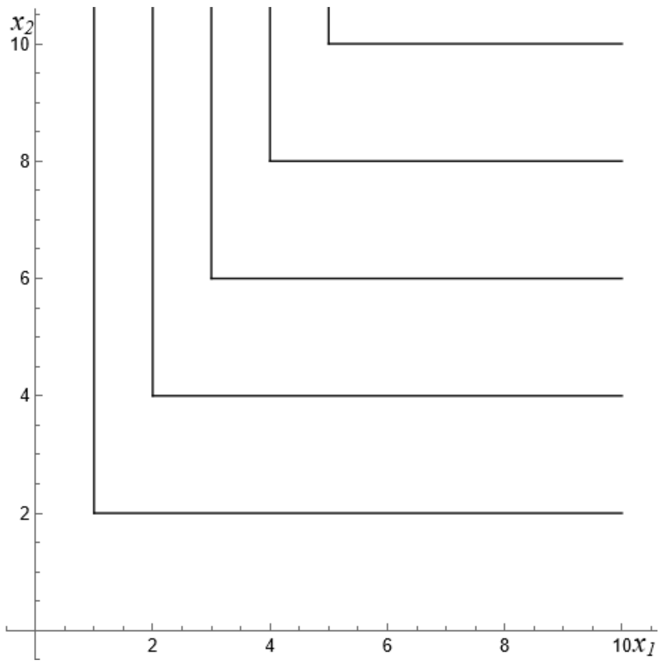

Figure 1: Baker's Isoquants

1.3 Technical Rate of Substitution

The most important thing about an indifference curve is it's slope. Because it represents a tradeoff:

Consumer's the **Marginal Rate of Substitution.** How much x_2 am I willing to give up if I consume one more unit of x_1.

Producer's the **Technical Rate of Substitution.** How much less x_2 I can use if I use one more unit of x_1.

$$TRS = -\frac{\frac{\partial(f(x_1,x_2))}{\partial x_1}}{\frac{\partial(f(x_1,x_2))}{\partial x_2}}$$

1.4 Marginal Products

$MP_1 = \frac{\partial(f(x_1,x_2))}{\partial x_1}$ marginal product of input 1.

Roughly: How many more units of output will I get if I increase x_1 by one unit?

$MP_2 = \frac{\partial(f(x_1,x_2))}{\partial x_2}$ marginal product of input 2.

Roughly: How many more units of output will I get if I increase x_2 by one unit?

$f(x_1, x_2) = x_1 x_2$

$$MP_1 = \frac{\partial\left(f\left(x_1, x_2\right)\right)}{\partial x_1} = \frac{\partial\left(x_1 x_2\right)}{\partial x_1} = x_2$$

$$MP_2 = \frac{\partial\left(f\left(x_1, x_2\right)\right)}{\partial x_1} = \frac{\partial\left(x_1 x_2\right)}{\partial x_1} = x_1$$

1.5 Diminishing Marginal Product

Diminishing marginal product is the idea that adding one input while holding the other constant will increase productivity less and less as the input you are adding increases.

If we have wokers and zero tools, the first tool will make them much more productive. The second tool will make more productive but productivity increases less than the extra amount from the first tool.

This is **Diminishing Marginal Product.**

To check whether there is diminishing marginal product for x_1 just check that $\frac{\partial(f(x_1,x_2))}{\partial x_1}$ is decreasing in x_1.

$$f\left(x_1, x_2\right) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

$$\frac{\partial\left(x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}\right)}{\partial x_1} = \frac{\sqrt{x_2}}{2\sqrt{x_1}}$$

$$\frac{\sqrt{x_2}}{2\sqrt{x_1}}$$

Decreasing in x_1 since x_1 is only in the denominator. If we wanted to see this formally, we need to check whether the second derivative negative.

Diminishing MP if decreasing in that good.

Constant MP if constant in that good.

Increasing MP if increasing in that good.

$$mp_1 = \frac{\partial\left(x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}\right)}{\partial x_1}$$

$$mp_1 = \frac{\sqrt{x_2}}{2\sqrt{x_1}}$$

$$\frac{\partial\left(mp_1\right)}{\partial x_1}=\frac{\partial\left(\frac{\sqrt{x_2}}{2\sqrt{x_1}}\right)}{\partial x_1}=-\frac{\sqrt{x_2}}{4x_1^{3/2}}$$

$$-\frac{\sqrt{x_2}}{4x_1^{3/2}}<0$$

Thus, $x_1^{\frac{1}{2}}x_2^{\frac{1}{2}}$ has diminishing marginal product for x_1.

Rule of thumb: for a cobb douglass production function, if the exponent is less than 1, the that production function has diminishing marginal product in that good.

$$x_1^{\frac{1}{2}}x_2^2$$

Diminishing marginal product of x_1 and increasing marginal product of x_2.

Thus, this is increasing marginal product.

$$\frac{\partial\left(x_1^{\frac{1}{2}}x_2^2\right)}{\partial x_2} = 2\sqrt{x_1}x_2$$

1.6 Returns to Scale

Measures additional outputs when we **scale** both inputs at the same time.

Let's take the example of:

$$f\left(x_1,x_2\right)=x_1x_2$$

Let's start at $f\left(1,1\right)=1$. If we double both inputs to $(2,2)$ we get four times the amount of output: $f\left(2,2\right)=4$

This is an example of **increasing returns to scale.**

Formally, this is the definition:

For $t>1$

Increasing returns to scale: $f\left(tx_1,tx_2\right)>tf\left(x_1,x_2\right)$

Constant returns to scale: $f\left(tx_1,tx_2\right)=tf\left(x_1,x_2\right)$

Decreasing returns to scale: $f\left(tx_1,tx_2\right)<tf\left(x_1,x_2\right)$

1.6.1 Example: Perfect Substitutes

$x_1 + x_2$

$$f(tx_1, tx_2) = (tx_1 + tx_2) = t(x_1 + x_2) = tf(x_1, x_2)$$

1.6.2 Another Example

$(x_1 + x_2)^2$

$$f(tx_1, tx_2) = (tx_1 + tx_2)^2 = (t(x_1 + x_2))^2 = t^2(x_1 + x_2)^2$$

For any $t > 1$

$$t^2(x_1 + x_2)^2 > t(x_1 + x_2)^2$$

Increasing returns to scale.

1.6.3 Rule of Thumb: Cobb Douglass

$$x_1^\alpha x_2^\beta$$

Increasing returns if $\alpha + \beta > 1$
Constant returns if $\alpha + \beta = 1$
Decreasing returns if $\alpha + \beta < 1$