

# 1 Monopoly Behavior

## 1.1 Bundling

This is possible when a firm sells more than one type of thing. Bundling is when the firm forces people to buy a bundle of those things rather than allowing them to buy each individual product.

*Cable Television, Microsoft Office*

	Shirt	Pants	Both
Consumer 1	50	30	80
Consumer 2	10	80	90

Price Shirts.

If the firm sets a price at \$10, both buy. Revenue is \$20.

If price at \$50, one buys and revenue is \$50.

Pricing Pants.

If the firm sets a price at \$30, both buy. Revenue is \$60.

If they set a price of \$80, one buys and revenue is \$80.

Total Revenue from selling separately is  $80 + 50 = \$130$ .

Pricing the Bundle.

**At a price of \$80, both buy and revenue is \$160.**

## 1.2 Two-Part Tariff

These are effective when consumers potentially buy more than one unit of a good.

For example, suppose each consumer's demand for coffee is  $q = 10 - p$ . The firm has zero cost for coffee.

**Standard Pricing:**

Profit function for coffee:

Inverse demand:  $p = 10 - q$

$$\pi(q) = q(10 - q)$$

$$\frac{\partial (q(10 - q))}{\partial q} = 10 - 2q$$

$$10 - 2q = 0$$

$$q = 5$$

$$p = 5$$

$$\pi = 25$$

Two part tariff. Charge your marginal cost per cup. In this case, charge  $p = 0$ . What is the most I can charge the consumer for the right to buy coffee at  $p = 0$ ? The most I can charge for this right is \$50. **This earns the firm \$50 profit.**

## 2 The Cournot Model of Competition

In the cournot model, there are  $N$  firms that each choose a quantity  $q_i$ . The goal of each firm is to maximize its profit by choosing its quantity, subject to the quantities chosen by the other firms.

Notation:

Firm  $i$ 's quantity:  $q_i$

Total quantity:  $Q = \sum_{i=1}^N q_i$

Total quantity from all firms except  $i$ :  $Q_{-i} = (Q - q_i)$

The price in the market is determined by the most that consumer are willing to spend to by  $Q$  units.

The price in the market is  $p(Q)$  where  $p()$  is the inverse demand function.

$$\pi_i(q_i, Q_{-i}) = q_i p(Q) - c(q_i)$$

$$Q_{-i} = Q - q_i. \quad Q = Q_{-i} + q_i$$

$$\pi_i(q_i, Q_{-i}) = q_i p(Q_{-i} + q_i) - c(q_i)$$

### 2.1 Example of Maximizing Profit with Two Firms

Suppose inverse demand is  $p(Q) = 100 - Q$ , there are two firms, and the cost function of each firm is  $c(q_i) = 10q_i$ .

$$\pi_1(q_1, q_2) = q_1(100 - (q_1 + q_2)) - 10q_1$$

$$\pi_2(q_2, q_1) = q_2(100 - (q_2 + q_1)) - 10q_2$$

### 2.1.1 Example firm 1 maximizing Profit

#### 2.1.2 50 Units for Firm 2

Suppose firm 2 is known to be producing 50 units  $q_2 = 50$

$$\pi_1(q_1, 50) = q_1(100 - (q_1 + 50)) - 10q_1$$

$$\pi_1 = q_1(100 - (q_1 + 50)) - 10q_1$$

$$\frac{\partial (q_1(100 - (q_1 + 50)) - 10q_1)}{\partial q_1} = 40 - 2q_1$$

$$20 = q_1$$

If firm 2 produces 50, firm 1 want's to produce 20.

#### 2.1.3 20 Units for Firm 2

$$\pi_1(q_1, 20) = q_1(100 - (q_1 + 20)) - 10q_1$$

$$\frac{\partial (q_1(100 - (q_1 + 20)) - 10q_1)}{\partial q_1} = 70 - 2q_1$$

$$q_1 = 35$$

## 2.2 Game Theory

Notice that firm 1's optimal decision depends on firm 2's decision and vice versa. This is the realm of game theory which is the study of **strategy**.

## 2.3 Best Responses

Firm 1's **Best Response** to  $q_2 = 50$  was  $q_1 = 20$

Firm 1's **Best Response** to  $q_2 = 20$  was  $q_1 = 35$

## 2.4 Equilibrium

Is both firm choosing 50 reasonable? In this case both firms have incentive to change to 20. **This is not an equilibrium.**

Is both choosing 20 a equilibrium? Either would have incentive to produce 35.

When it is the case that a pair of quantities are both best responses to the quantities chosen by the other, we say the game is in **Nash Equilibrium.**

**Nash Equilibrium:** A set of mutual best response.

## 2.5 Finding a Nash Equilibrium

Write down the **best response functions.**

$$\pi_1(q_1, q_2) = q_1(100 - (q_1 + q_2)) - 10q_1$$

$$\pi_2(q_2, q_1) = q_2(100 - (q_2 + q_1)) - 10q_2$$

Let's find optimal  $q_1$  for any  $q_2$ .

$$\frac{\partial (q_1(100 - (q_1 + q_2)) - 10q_1)}{\partial q_1} = -2q_1 - q_2 + 90$$

$$-2q_1 - q_2 + 90 = 0$$

$$90 - q_2 = 2q_1$$

Firm 1's best response:

$$q_1 = \frac{90 - q_2}{2}$$

Firm 2's best response:

$$\frac{\partial (q_2(100 - (q_2 + q_1)) - 10q_2)}{\partial q_2} = -q_1 - 2q_2 + 90$$

$$-q_1 - 2q_2 + 90 = 0$$

$$\frac{90 - q_1}{2} = q_2$$

$$q_2 = \frac{90 - q_1}{2}$$

Equilibrium is a set of mutual best responses.  $q_1, q_2$  such that  $q_1$  is a best response to  $q_2$  and  $q_2$  is a best response to  $q_1$ . Solve these simultaneously:

$$q_1 = \frac{90 - q_2}{2}$$

$$q_1 = 30, q_2 = 30$$

## 2.6 Symmetric Equilibrium

$$q = \frac{90 - q}{2}$$

Solve this for  $q$  :

$$q = \frac{90 - q}{2}$$

$$2q = 90 - q$$

$$3q = 90$$

$$q = 30$$