0.1 Cournot Example from Exam

Market Demand is $200 - 40p$ cost if $c(q) = q$

$$Q = 200 - 40p$$

$$40p = 200 - Q$$

Inverse demand:

$$p = \frac{200}{40} - \frac{1}{40}Q$$

Profit function for a monopolist:

$$\pi(q) = q\left(\frac{200}{40} - \frac{1}{40}q\right) - q$$

Maximize by finding where the slope is zero:

$$q\left(\frac{200}{40} - \frac{1}{40}q\right) - q$$

$$q\frac{200}{40} - \frac{1}{40}q^2 - q$$

$$= 4q - \frac{1}{40}q^2$$

$$\frac{\partial\left(4q - \frac{1}{40}q^2\right)}{\partial q} = 0$$

$$4 - \frac{1}{20}q = 0$$

Monopoly chooses:

$$q^* = 80$$

$$\pi(80) = 4(80) - \frac{1}{40}(80)^2 = 320 - 160 = 160$$

D)

$$\pi_1(q_1, q_2) = q_1\left(\frac{200}{40} - \frac{1}{40}Q\right) - q_1$$

$$= q_1\left(\frac{200}{40} - \frac{1}{40}(q_1 + q_2)\right) - q_1$$

E)

$$\frac{200}{40}q_1 - \frac{1}{40}q_1^2 - \frac{1}{40}q_1q_2 - q_1$$

$$= 4q_1 - \frac{1}{40}q_1^2 - \frac{1}{40}q_1q_2$$

Take the derivative and find where it is zero:

$$4q_1 - \frac{1}{40}q_1^2 - \frac{1}{40}q_1q_2$$

$$\frac{\partial\left(4q_1 - \frac{1}{40}q_1^2 - \frac{1}{40}q_1q_2\right)}{\partial q_1} = 0$$

$$4 - \frac{2}{40}q_1 - \frac{1}{40}q_2 = 0$$

Solve this for q_1:

$$4 - \frac{1}{40}q_2 = \frac{2}{40}q_1$$

$$\frac{40}{2}4 - \frac{40}{2}\frac{1}{40}q_2 = q_1$$

$$q_1 = 80 - \frac{1}{2}q_2$$

A Nash equilibrium is a pair of quantities such that both firms are best responding at the same time.
For either firm i:

$$q_i = 80 - \frac{1}{2}q_j$$

To solve for NE we look a $q = q_1 = q_2$ that solves any of the best responses.

$$q = 80 - \frac{1}{2}q$$

$$\frac{3}{2}q = 80$$

$$q^* = \frac{160}{3}$$

$$Q = 2\frac{160}{3}$$

$$p = \frac{7}{3}$$

Find the collusive quantity. We can look for a q that maximizes any one of the firm's profits.

$$\pi_1 = q\left(\frac{200}{40} - \frac{1}{40}(q+q)\right) - q$$

This is maximized where it's slope is zero.
Slope

$$4 - \frac{q}{10} = 0$$

Solve for q :

$$4 = \frac{q}{10}$$

$$q = 40$$

0.2 Production Example from Exam:

$$f(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{1}{3}}$$

For a $t > 1$

$$f(tx_1, tx_2) < tf(x_1, x_2)$$

To check this:

$$(tx_1)^{\frac{1}{3}}\,(tx_2)^{\frac{1}{3}}$$

$$= t^{\frac{1}{3}} x_1^{\frac{1}{3}} t^{\frac{1}{3}} x_2^{\frac{1}{3}}$$

$$= t^{\frac{2}{3}} \left(x_1^{\frac{1}{3}} x_2^{\frac{1}{3}} \right)$$

Decreasing returns to scale.

$$-\frac{w_1}{w_2} = -TRS$$

$$-\frac{4}{1} = -\frac{\frac{\partial\left(x_1^{\frac{1}{3}} x_2^{\frac{1}{3}}\right)}{\partial x_1}}{\frac{\partial\left(x_1^{\frac{1}{3}} x_2^{\frac{1}{3}}\right)}{\partial x_2}}$$

$$-\frac{4}{1} = -\frac{\frac{\partial\left(x_1^{\frac{1}{3}} x_2^{\frac{1}{3}}\right)}{\partial x_1}}{\frac{\partial\left(x_1^{\frac{1}{3}} x_2^{\frac{1}{3}}\right)}{\partial x_2}}$$

$$-\frac{\frac{1}{3} x_1^{-\frac{2}{3}} x_2^{\frac{1}{3}}}{\frac{1}{3} x_2^{-\frac{2}{3}} x_1^{\frac{1}{3}}} = -\frac{x_2}{x_1}$$

$$-\frac{4}{1} = -\frac{x_2}{x_1}$$

Two conditions for optimality:

$$4x_1 = x_2$$

$$x_1^{\frac{1}{3}} x_2^{\frac{1}{3}} = y$$

$$x_1^{\frac{1}{3}} (4x_1)^{\frac{1}{3}} = y$$

$$4^{\frac{1}{3}}x_1^{\frac{1}{3}}x_1^{\frac{1}{3}} = y$$

$$x_1^{\frac{2}{3}} = \frac{y}{4^{\frac{1}{3}}}$$

$$x_1 = \left(\frac{y}{4^{\frac{1}{3}}}\right)^{\frac{3}{2}} = \frac{y^{\frac{3}{2}}}{2} = \frac{1}{2}y^{\frac{3}{2}}$$

$$x_2 = 2y^{\frac{3}{2}}$$

$$c\left(y\right) = 4\frac{1}{2}y^{\frac{3}{2}} + 2y^{\frac{3}{2}} = 4y^{\frac{3}{2}}$$