1 Cournot

1.1 Nash Equilibrium with Two Firms

Suppose inverse demand is $p(Q) = 100 - Q$, there are two firms, and the cost function of each firm is $c(q_i) = 10q_i$.

$$\pi(q_i, Q_{-i}) = q_i(100 - q_i - Q_{-i}) - 10q_i$$

$$\frac{\partial(q_i(100 - q_i - Q_{-i}) - 10q_i)}{\partial q_i}$$

$$\frac{\partial\left(q_i 100 - q_i^2 - q_i Q_{-i} - 10q_i\right)}{\partial q_i} = 100 - 2q_i - Q_{-i} - 10$$

$$= 90 - 2q_i - Q_{-i}$$

$$90 - 2q_i - Q_{-i} = 0$$

Best Response:

$$q_i = \frac{90 - Q_{-i}}{2}$$

If there are only two firms q_1, q_2

$$q_1 = \frac{90 - q_2}{2}$$

$$q_2 = \frac{90 - q_1}{2}$$

Nash equilibrium occurs where both firms choose a quantity that is a best response to the other firm's quantity.

$q_1 = 30, q_2 = 50$

Is 30 a best response to 50?

$$30 = \frac{90 - 50}{2}$$

$$30 = 20$$

30 isn't a best response to 50. What is the best response? 20 is.

$$20 = \frac{90 - 50}{2}$$

We need to find a q_1 and q_2 that are mutual best responses.

$$q_1 = \frac{90 - q_2}{2}$$

$$q_2 = \frac{90 - q_1}{2}$$

Plug q_2 into q_1 :

$$q_1 = \frac{90 - \left(\frac{90 - q_1}{2}\right)}{2} = \frac{90 - 45 + \frac{1}{2}q_1}{2}$$

$$q_1 = \frac{45}{2} + \frac{1}{4}q_1$$

$$\frac{3}{4}q_1 = \frac{45}{2}$$

$$q_1 = 30$$

Plug this back into best response for q_2

$$q_2 = \frac{90 - 30}{2} = \frac{60}{2} = 30$$

The only nash equilibrium is $(30, 30)$.
This is a symmetric nash equilibrium, because both firms choose the same quantity.

$$q_1 = q_2 = q$$

Suppose we limit our search for equilibrium to symmetric nash equilibrium.

$$q = \frac{90 - q}{2}$$

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When we impose symmetry, there is now only one equation to solve:

$$q = \frac{90 - q}{2}$$

$$2q = 90 - q$$

$$3q = 90$$

$$q = 30$$

1.2 Equilibrium with N firms.

This is the generic best response function:

$$q_i = \frac{90 - Q_{-i}}{2}$$

Let's suppose all firms choose $q_i = q$

$$Q = \sum_{i=1}^{n} q = nq$$

$$Q_{-i} = Q - q_i = nq - q = (n-1)\,q$$

$$q = \frac{90 - (n-1)\,q}{2}$$

$$2q = 90 - (n-1)\,q$$

$$(n-1)\,q + 2q = 90$$

$$(n+1)\,q = 90$$

$$q = \frac{90}{n+1}$$

For 2 firms:

$$q = \frac{90}{3} = 30$$

Firm 9 firms:

$$q = \frac{90}{10} = 9$$

Market quantity:

$$Q = nq = \frac{n}{n+1} 90$$

Market price (inverse demand evalutaed at Q)

$$p(Q) = 100 - \frac{n}{n+1} 90$$

Profit of each firm:

$$\pi = \frac{90}{n+1}\left(100 - \frac{n}{n+1}90\right) - 10\left(\frac{90}{n+1}\right)$$

Let's look at these for various N:

$$\begin{pmatrix} 2. & 30. & 60. & 40. & 900. \\ 3. & 22.5 & 67.5 & 32.5 & 506.25 \\ 5. & 15. & 75. & 25. & 225. \\ 10. & 8.18182 & 81.8182 & 18.1818 & 66.9421 \\ 100. & 0.891089 & 89.1089 & 10.8911 & 0.79404 \\ 1000. & 0.0899101 & 89.9101 & 10.0899 & 0.00808382 \end{pmatrix}$$

In perfect competiion price=marginal cost. $mc = 10$. $c(q) = 10q$
For a monopolist:

$$\frac{\partial (q(100-q) - 10q)}{\partial q} = 90 - 2q$$

$$45 = q$$

This is also the solution we get in the cournot model when we set $n = 1$.

2 Collusion and Cooperation

Suppose inverse demand is $p(Q) = 100 - Q$, there are two firms, and the cost function of each firm is $c(q_i) = 10q_i$.

$$\pi(q_1, q_2) = q_1(100 - q_1 - q_2) - 10q_1$$

$$\pi(q_2, q_1) = q_2(100 - q_1 - q_2) - 10q_2$$

In equilibrium, both firms choose $q = 30$ and $Q = 60$. $\pi = 30(100 - 60) - 10(30) = 900$

Suppose the firms agree to maximzie their profits by choosing a non-equilibrium q:

$$\pi(q, q) = q(100 - q - q) - 10q$$

$$\frac{\partial(q(100 - q - q) - 10q)}{\partial q} = 90 - 4q$$

$$\frac{90}{4} = q$$

$$q = 22.5$$

$$p(Q) = 100 - 2(22.5) = 55.$$

$$\pi = 22.5(55) - 10(22.5) = 1012.5$$

They earn 112.5 more than in equilibrium.

This is an equilibrium, either firm can earn even more by best responding with:

$$q = 33.75$$

2.1 A Basic Example

2.2 Prisoner's Dilemma

	cooperate	defect
cooperate	4,4	0,**10**
defect	**10**,0	**2**,**2**

Figure 1: A simplified prisoner's dilemma game.