0.1 Prisoner's Dilemma

	cooperate	defect
cooperate	4,4	0,**10**
defect	**10**,0	**2**,**2**

Figure 1: A simplified prisoner's dilemma game.

Pareto Efficient: No one can be made better off without someone being worse off.

In prisoner's dilemma, the only Nash equilibrium is not pareto efficient.

0.2 Sustaining Cooperation

Suppose the players play this game every day forever. **Infinitely Repeated Game. Each discounts the next day β (discount factor) from the day before.**

$$\beta = 0.5$$

"I care about tomorrow half as much as I care about today"

$$\beta = 0.99$$

"I care about tomorrow *almost as much* as much as I care about today"

Suppose both players play the nash equilibrium forever.

$$2 + 2\beta + 2\beta^2 + 2\beta^3 + ...$$

$$= \sum_{t=0}^{\infty} 2\beta^t = 2\left(\sum_{t=0}^{\infty} \beta^t\right)$$

Here's a math trick (for $0 < \beta < 1$)

$$\sum_{t=0}^{\infty} \beta^t = \frac{1}{1-\beta}$$

The infinite stream of 2's is worth:

$$2 + 2\beta + 2\beta^2 + 2\beta^3 + ... = \frac{2}{1-\beta}$$

$\beta = 0.5$

$$\frac{2}{1-0.5} = \frac{2}{0.5} = 4$$

$\beta = 0.99$

$$\frac{2}{1-0.99} = 200$$

Compare this to the strategy that leads of us to cooperate forever:

$$\sum_{t=0}^{\infty} 4\beta^t = \frac{4}{1-\beta}$$

"Grim Trigger Strategy": Cooperate as long as no one has ever defected.

Let's check that this is a Nash equilibrium. If you go along with the strategy:

$$4 + 4\beta + 4\beta^2 + ...$$

$$\sum_{t=0}^{\infty} 4\beta^t = \frac{4}{1-\beta}$$

If I am ever going to deviate from this strategy, it will lead us to both defect forever after I deviate.

If I defect, I get 10 today

$$10 + 2\beta + 2\beta^2 + 2\beta^3 + ... = 10 + \beta\left(2 + 2\beta + 2\beta^2 + ...\right)$$

$$= 10 + \beta\frac{2}{1-\beta}$$

Is it worth defecting? It is not worth defecting, and thus cooperating is self-enforcing:

$$\frac{4}{1-\beta} > 10 + \beta\frac{2}{1-\beta}$$

$$\frac{4}{1-\beta} - \beta\frac{2}{1-\beta} > 10$$

$$\frac{4}{1-\beta} - \frac{2\beta}{1-\beta} > 10$$

$$\frac{4-2\beta}{1-\beta} > 10$$

$$4-2\beta > (1-\beta)\,10$$

$$4-2\beta > 10-10\beta$$

$$10\beta + 4 - 2\beta > 10$$

$$8\beta > 6$$

$$\beta > \frac{3}{4}$$

As long as I care 75% about tomorrow as much as I care about today, then it is not worth defecting from cooperation.

1 Externalities

Externatlies occur when someones actions hurts or helps another person, but the way that the first persons actions affect other's utility are not taken into account when optimizing.

Negative Externalities: Second-hand smoke. Pollution.

Positive Externalities: Mowing my grass makes the whole block look nicer which might improve home values.

1.1 Tragedy of the Commons Example

Fisheries. When I catch a fish, it reduces the density of fish in the lake and makes it harder for others to catch fish.

Example:

B boats go the lake.

Suppose $100\sqrt{B}$ fish will be caught on a lake when B boats are fishing.

Fish can be sold for $1 but it costs $10 to buy fuel and supplies to fish.

When is it in my interest to fish?

It will be in every boats interest to remain the lake as long as they earn positive profit from fishing.

What is the profit per boat?

Fish caught per boat:

$$\frac{100\sqrt{B}}{B}$$

Revenue per boat (fish caught per boat time price per fish $1):

$$\left(\frac{100\sqrt{B}}{B}\right)$$

Profit:

$$\left(\frac{100\sqrt{B}}{B}\right) - 10$$

When is best to stay on the lake?

$$\left(\frac{100\sqrt{B}}{B}\right) - 10 \geq 0$$

$$\frac{100\sqrt{B}}{B} \geq 10$$

$$100\sqrt{B} \geq 10B$$

$$10 \geq \sqrt{B}$$

$$100 \geq B$$

Individually optimal level of boats.

What is the socially optimal number of boats?

Imagine one company owns the right to the lake. Their goal is to maximize profit.

Total profit earned all boats on the lake:

$$B\left(\left(\frac{100\sqrt{B}}{B}\right) - 10\right)$$

$$100\sqrt{B} - 10B$$

To find the optimal number of boats (with respect to total profit):

$$\frac{\partial\left(100\sqrt{B} - 10B\right)}{\partial B} = \frac{50}{\sqrt{B}} - 10$$

$$\frac{50}{\sqrt{B}} - 10 = 0$$

$$\frac{50}{\sqrt{B}} = 10$$

$$\sqrt{B} = 5$$

$$B = 25$$

Total profit:

$$= 250$$

1.2 Optimal Fee For Fishing.