Functions.

Functions have Independent variables: x and dependent variables: y. They are a "Functional Relationship" $y = f(x)$. Think of f as a recipe.

$y = f(x) = x + 2$
$y = f(x) = x^2$

Properties of functions:
Roots: for what values does $y = 0$?
Maximum, Minima: for what values of x what values is y highest? lowest?

How do we find roots? Usually it's easy, just set $y = 0$ and solve for x...

$y = x^2$. $0 = x^2$. $0 = x$.

$y = x^2 - 1$. $1 = x^2$. $x = -1\, or\, 1$.

Sometimes it's not so trivial. Suppose you have something like: $y = ax^2 + bx + c$. Then the roots are given by $\frac{-b \pm \sqrt{(b^2-4ac)}}{2a}$: the "quadratic formula".

Why do we want to find roots anyway? They help us find maxima and minima, which are things we care about a lot in economics.

A maxima or minima can't happen when the function is changing in value! Think about his for a moment. If you want a maximum, or a minimum, you have to look for a "stationary" point. A point where the slope is zero. Recall from calculus that the derivative of a function gives your the slope of the function at that point. If we can take derivatives, we can find stationary points and then we know where to look for maxima or minima.

Here are some useful rules for derivatives.

Power rule:
$y = ax^b$
Derivative: abx^{b-1}

For example $2x^3$ has derivative $6x^2$ and the derivative of x^2 is $2x$ and the derivative of $3x$ is $3x^0 = 3$. This extends to negatives as well. The derivative of $\frac{2}{x}$=$2x^{-1}$ is $-2x^{-2} = -\frac{2}{x^2}$.

Derivative of a sum is the sum of the derivatives.
For example:

$2x^3 + x^2$ has derivative $6x^2 + 2x$.

Derivative of a constant is 0.

It is also useful to remember that the derivative of $ln\left(x\right) = \frac{1}{x}$ and that the derivative of e^x is e^x.

Product Rule:

The derivative of a product is the derivative of the first element times the second plus the derivative of the second element times the first. For instance the derivative of:

$$2x^2 ln\left(x\right)$$

Is:

$$4x ln\left(x\right) + 2x^2 \frac{1}{x} = 4x ln\left(x\right) + 2x$$

The derivative of:

$$\left(2x^3 + 2x\right)\left(3x^2 + 4x^4\right)$$

Is:

$$\left(6x^2 + 2\right)\left(3x^2 + 4x^4\right) + \left(2x^3 + 2x\right)\left(6x + 16x^3\right)$$

Chain Rule:

This will help us do some complex derivatives. In general, if we have:

$$g\left(f\left(x\right)\right)$$

It's derivative is:

$$g'\left(f\left(x\right)\right)f'\left(x\right)$$

Suppose we have:

$$ln\left(6x^2 + 2\right)$$

What should we do with this? $g\left(\right)$ is $ln\left(z\right)$ which has derivative $\frac{1}{z}$. Here, we've simply called $6x^2 + 2$ by the name z. Thus, the $g'\left(f\left(x\right)\right)$ part is: $\frac{1}{6x^2+2}$. Now, the $f'\left(x\right)$ is simply the derivative of $6x^2 + 2$ which is $12x$. We have the following derivative:

$$\frac{1}{6x^2 + 2}12x = \frac{12x}{6x^2 + 2}$$

We can use this is other ways. Consider the following function:

$$y = \frac{1}{6x^2+2} = \left(6x^2+2\right)^{-1}$$

We know the derivative of the inside using the power rule and the fact that the derivative of a sum is the sum of derivatives. We also know how to take the derivative of something like z^{-1} which is $-z^{-2}$. Let's apply the chain rule to this function:

$$g'\left(f\left(x\right)\right) - \left(6x^2+2\right)^{-2}$$

$$f'\left(x\right) = 12x$$

Putting this together, the derivative of $-\left(6x^2+2\right)^{-2}\left(12x\right)$ is

$$-\frac{12x}{\left(6x^2+2\right)^2}$$

Often, people memorize what is called the "quotient rule" which says that the derivative of a quotient: $\frac{f(x)}{g(x)}$ is $\frac{g(x)f'(x)+f(x)g'(x)}{(g(x))^2}$. You can get away with not having this memorized because the same thing can be achieved by applying both the chain and product rule when you note that:

$$\frac{f\left(x\right)}{g\left(x\right)} = f\left(x\right)\left(g\left(x\right)\right)^{-1}$$

However, in most cases, it is much easier to simply apply this "quotient rule".

For instance the derivative of $\frac{ln(x)}{x^2}$ is

$$\frac{x^2\frac{1}{x} + 2xln\left(x\right)}{\left(x^2\right)^2} = \frac{x + 2xln\left(x\right)}{x^4} = \frac{1+2ln\left(x\right)}{x^3}$$

Partial Derivatives:

When doing partial derivatives of multivariate functions, we treat all but the variable we are taking the derivative with respect to as if it is a constant.

For instance, partial derivative of $3x^2 + 2z^4$ with respect to x is simply $6x$. The partial derivative of $3x^2z + 2x + 8z$ with respect to x is $6xz + 2$.