

Econ 3012 - Midterm Exam Solution

April 24, 2023

1. Briefly describe the following in a way that a person who **has not studied economics or mathematics would understand**:

A. What is the **marginal rate of substitution**?

The marginal rate of substitution measures a consumer's willingness to trade off between two goods. It measures approximately how much of one good they will give up to get one unit of the other good.

B. What are **homothetic** preferences?

Homothetic preferences are preferences where a consumer's willingness to trade off between two goods only depends on the ratio of the goods they consuming but not the absolute value.

C. What is a **utility function**?

A utility function is a numerical representation of a consumer's preferences with the property that if one bundle of goods is preferred to another, the more preferred bundle has a higher utility.

2. Fill in the blank.

A. Suppose we know the following about a consumer's preferences over the bundles $\{a, b, c\}$: $a \sim a$, $b \sim b$, $c \sim c$, $a \succ b$, $b \succ c$, $c \succ a$. These preferences are not **Transitive**.

B. If demand is elastic, a 1% increase in price will lead to **more than a 1% decrease in demand**.

C. The demand for one good goes up when the price of another goes up. These goods are **substitutes**.

3. A consumer's utility function is $x_1 + \sqrt{x_2}$. Prices are $p_1 = 2$, $p_2 = 1$, $m = 100$.

A) Are the preferences of this consumer monotonic?

They are since utility is increasing in both x_1 and x_2 .

B) What is the consumer's marginal rate of substitution?

$$MRS = -\frac{\frac{\partial(x_1 + \sqrt{x_2})}{\partial x_1}}{\frac{\partial(x_1 + \sqrt{x_2})}{\partial x_2}} = -2\sqrt{x_2}$$

C) What amount of x_2 does the consumer demand at these prices?

$$x_2 = 1$$

D) What is the consumer's price elasticity of demand for x_2 when $p_1 = 2$, and $m = 100$?

$$MRS = -\frac{p_1}{p_2}$$

Demand for x_2 is: $x_2 = \frac{1}{p_2^2}$

$$\varepsilon = \frac{\partial\left(\frac{1}{p_2^2}\right)}{\partial p_2} \frac{p_2}{\frac{1}{p_2^2}} = -2$$

4. A consumer has an endowment of $\omega_1 = 12, \omega_2 = 6$. Their utility function is $u(x_1, x_2) = x_1 x_2$. Prices are $p_1 = 1, p_2 = 1$

A) What is this consumer's budget equation?

$$p_1 x_1 + p_2 x_2 = p_1 \omega_1 + p_2 \omega_2$$

$$x_1 + x_2 = 18$$

B) Are the preferences of this consumer homothetic?

Yes, notice that the MRS is $\frac{x_2}{x_1}$ which only depends on the ratio of the goods.

C) What bundle does this consumer demand at these prices?

$$x_1 = 9, x_2 = 9$$

D) Suppose $p_1 = 1$. What would p_2 need to be so that this consumer is neither a net-buyer nor a net-seller of x_2 ?

Budget equation:

$$x_1 + p_2 x_2 = 12 + 6p_2$$

Equal-Slope Condition:

$$-\frac{x_2}{x_1} = -\frac{1}{p_2}$$

Solving these:

$$x_1 = 3(p_2 + 2), x_2 = \frac{3(p_2 + 2)}{p_2}$$

Now find the p_2 such that $x_1 = 12$ or find the p_2 such that $x_2 = 6$. In either case, we get: $p_2 = 2$.

5. A consumer has demand $x_1 = \frac{\frac{1}{2}m}{p_1}$ and $x_2 = \frac{\frac{1}{2}m}{p_2}$

A) Are these goods ordinary or Giffen?

Ordinary.

B) At $p_1 = 1$, $p_2 = 1$ and $m = 40$, what is this consumer's demand? What about if p_1 increases to $p_1 = 2$.

$$x_1 = 20, x_2 = 20$$

$$x_1 = 10, x_2 = 20$$

C) Of the change in demand for x_1 in part B, how much is due to the substitution effect?

Solve for the consumer's demand if they had enough money to buy the original bundle (20, 20) (this would take $m = 60$) and the new prices of $p_1 = 2, p_2 = 1$. With this budget they could consumer $x_1 = 15$. **Thus, 5 units is due to the substitution effect.**

D) Of the change in demand for x_1 in part B, how much is due to the income effect?

The remainder is due to the income effect. This is 5 units as well.