HW 7

2.1 a.) $10x_1 + 5x_2 = 40$

b.) 4

c.) 8

d.) $5x_1 + 5x_2 = 40$

e.) $5x_1 + 5x_2 = 30$

2.3 b.) $1/2$

c.) 16

d.) 8

e.) $x + 2y = 16$

f.) $3x + 6y = 48$

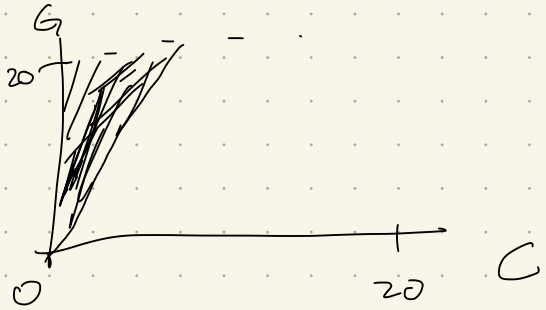
2.5 6

2.7 a.) 0

b.) 5

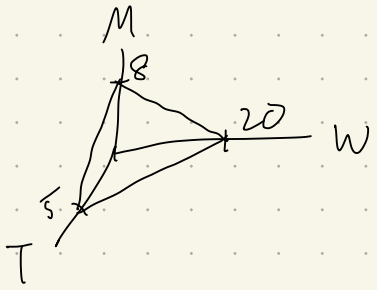
c.) $6C - 2G = 0$

d.)



2.9 a.) $20W + 80T + 50M = 400$

b.)



c.) $20W + 50M = 320$

2.11 a.) 10,000 / 3,000

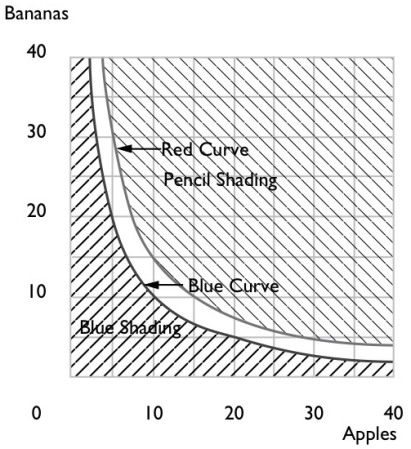
b.) 6000 / 5000

c.) 8000 / 4000

d.) No

e.) $M + 2L = 16$

3.1



For each of the following statements about Charlie's preferences, write "true" or "false."

(c) $(30, 5) \sim (10, 15)$. True.

(d) $(10, 15) \succ (20, 5)$. True.

(e) $(20, 5) \succeq (10, 10)$. True.

(f) $(24, 4) \succeq (11, 9.1)$. False.

(g) $(11, 14) \succ (2, 49)$. True.

(h) A set is convex if for any two points in the set, the line segment between them is also in the set. Is the set of bundles that Charlie weakly prefers to $(20, 5)$ a convex set? Yes.

(i) Is the set of bundles that Charlie considers inferior to $(20, 5)$ a convex set? No.

(j) The slope of Charlie's indifference curve through a point, (x_A, x_B), is known as his marginal rate of substitution at that point.

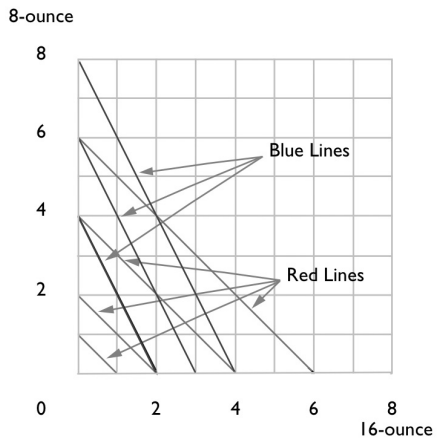
(k) Remember that Charlie's indifference curve through the point $(10, 10)$ has the equation $x_B = 100/x_A$. Those of you who know calculus will remember that the slope of a curve is just its derivative, which in this case is $-100/x_A^2$. (If you don't know calculus, you will have to take our word for this.) Find Charlie's marginal rate of substitution at the point, $(10, 10)$. -1.

(l) What is his marginal rate of substitution at the point $(5, 20)$? -4.

(m) What is his marginal rate of substitution at the point $(20, 5)$? $(-.25)$.

(n) Do the indifference curves you have drawn for Charlie exhibit diminishing marginal rate of substitution? Yes.

3.3

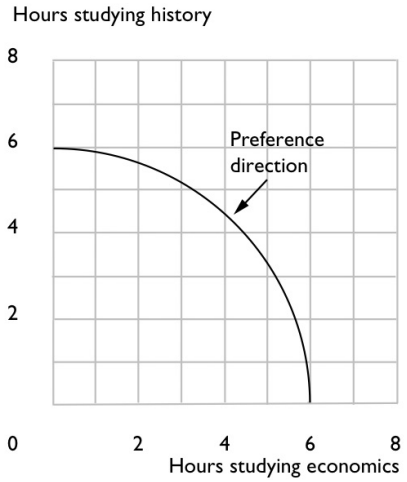


3.5

Randy has strictly convex preferences.

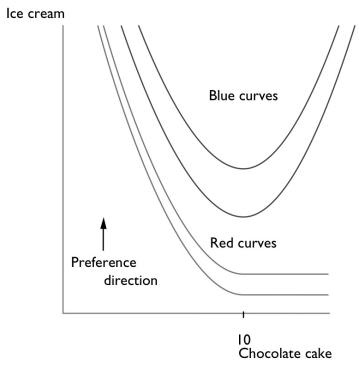
(a) Sketch an indifference curve for Randy where the two commodities are hours per week spent studying economics and hours per week spent studying history. Will the slope of an indifference curve be positive or negative? `Negative.`

(b) Do Randy's indifference curves get steeper or flatter as you move from left to right along one of them? `Steeper.`



3.6 (0) Flossy Toothsome likes to spend some time studying and some time dating. In fact her indifference curves between hours per week spent studying and hours per week spent dating are concentric circles around her favorite combination, which is 20 hours of studying and 15 hours of

3.7



3.13

(a) Does Steroid prefer Westinghouse to Hotpoint or vice versa? He prefers Westinghouse to Hotpoint.

(b) Does Steroid prefer Hotpoint to Jacuzzi or vice versa? He prefers Hotpoint to Jacuzzi.

(c) Does Steroid prefer Westinghouse to Jacuzzi or vice versa? He prefers Jacuzzi to Westinghouse.

(d) Does Coach Steroid have transitive preferences? No.

(e) After several losing seasons, Coach Steroid decides to change his way of judging players. According to his new preferences, Steroid prefers player A to player B if player A is better in all three of the characteristics that Steroid values, and he prefers B to A if player B is better at all three things. He is indifferent between A and B if they weigh the same, are equally fast, and are equally obedient. In all other cases, Coach Steroid simply says "A and B are not comparable."

(f) Are Coach Steroid's new preferences complete? No.

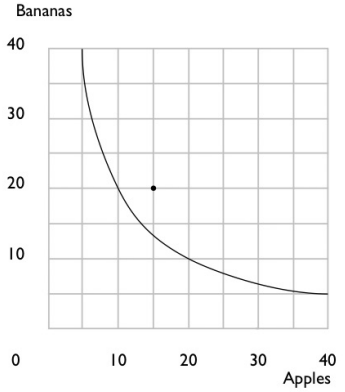
(g) Are Coach Steroid's new preferences transitive? Yes.

h) yes

4.1

to be $U(x_A, x_B) = x_A x_B$.

(a) Charlie has 40 apples and 5 bananas. Charlie's utility for the bundle $(40, 5)$ is $U(40, 5) =$ 200. The indifference curve through $(40, 5)$ includes all commodity bundles (x_A, x_B) such that $x_A x_B =$ 200. So the indifference curve through $(40, 5)$ has the equation $x_B =$ $\frac{200}{x_A}$. On the graph below, draw the indifference curve showing all of the bundles that Charlie likes exactly as well as the bundle $(40, 5)$.



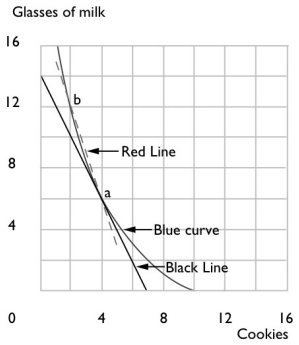
(b) Donna offers to give Charlie 15 bananas if he will give her 25 apples. Would Charlie have a bundle that he likes better than $(40, 5)$ if he makes this trade? Yes. What is the largest number of apples that Donna could demand from Charlie in return for 15 bananas if she expects him to be willing to trade or at least indifferent about trading? 30. (Hint: If Donna gives Charlie 15 bananas, he will have a total of 20 bananas. If he has 20 bananas, how many apples does he need in order to be as well-off as he would be without trade?)

4.3

consumes.

(a) What is the slope of Burt's indifference curve at the point where he is consuming the bundle $(4,6)$? $-2.$ Use pencil or black ink to draw a line with this slope through the point $(4,6)$. (Try to make this graph fairly neat and precise, since details will matter.) The line you just drew is the *tangent line* to the consumer's indifference curve at the point $(4,6)$.

(b) The indifference curve through the point $(4,6)$ passes through the points (10 ,0), (7, 2), and (2, 12). Use blue ink to sketch in this indifference curve. Incidentally, the equation for Burt's indifference curve through the point $(4,6)$ is $x_2 =$ $72/(x_1+2)-6.$



(c) Burt currently has the bundle $(4,6)$. Ernie offers to give Burt 9 glasses of milk if Burt will give Ernie 3 cookies. If Burt makes this trade, he would have the bundle $(1,15).$ Burt refuses to trade. Was this a wise decision? Yes, $U(1,15) = 63 < U(4,6) = 72.$ Mark the bundle $(1,15)$ on your graph.

(d) Ernie says to Burt, "Burt, your marginal rate of substitution is -2. That means that an extra cookie is worth only twice as much to you as an extra glass of milk. I offered to give you 3 glasses of milk for every cookie you give me. If I offer to give you more than your marginal rate of substitution, then you should want to trade with me." Burt replies,

"Ernie, you are right that my marginal rate of substitution is -2. That means that I am willing to make *small* trades where I get more than 2 glasses of milk for every cookie I give you, but 9 glasses of milk for 3 cookies is too big a trade. My indifference curves are not straight lines, you see." Would Burt be willing to give up 1 cookie for 3 glasses of milk? Yes, $U(3,9) = 75 > U(4,6) = 72.$ Would Burt object to giving up 2 cookies for 6 glasses of milk? No, $U(2,12) = 72 = U(4,6).$

(e) On your graph, use red ink to draw a line with slope -3 through the point $(4,6)$. This line shows all of the bundles that Burt can achieve by trading cookies for milk (or milk for cookies) at the rate of 1 cookie for every 3 glasses of milk. Only a segment of this line represents trades that make Burt better off than he was without trade. Label this line segment on your graph AB.

4.5

spectively. $U(x_1,x_2) =$ $\min\{x_1,x_2\}$, or any monotonic transformation.

4.9

(a) $f(u) = 3.141592u$. Yes.

(b) $f(u) = 5,000 - 23u$. No.

(c) $f(u) = u - 100,000$. Yes.

(d) $f(u) = \log_{10} u$. Yes.

(e) $f(u) = -e^{-u}$. Yes.

(f) $f(u) = 1/u$. No.

(g) $f(u) = -1/u$. Yes.

1

A.) $MRS = -\frac{x_2}{x_1}$

B.) $-\frac{3x_2}{x_1}$

C.) $-\frac{x_2}{x_1}$

D.) $-\frac{2x_1}{3}$

E.) $-\frac{x_2}{x_1}$

F.) $-\frac{1-x_2}{x_1}$

(A), (C), (E) represent the same preferences.

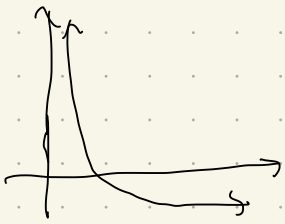
2 (Roughly).

A.)

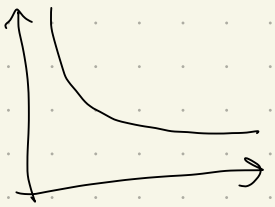
B.)

C.)

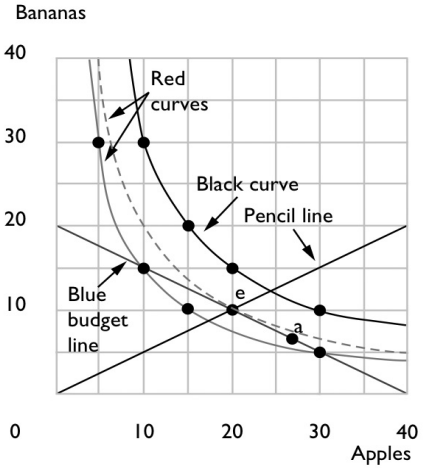
D.)



E.)



5.1



(b) Can Charlie afford any bundles that give him a utility of 150? Yes.

(c) Can Charlie afford any bundles that give him a utility of 300? No.

(d) On your graph, mark a point that Charlie can afford and that gives him a higher utility than 150. Label that point A.

(e) Neither of the indifference curves that you drew is tangent to Charlie's budget line. Let's try to find one that is. At any point, (x_A, x_B), Charlie's marginal rate of substitution is a function of x_A and x_B. In fact, if you calculate the ratio of marginal utilities for Charlie's utility function, you will find that Charlie's marginal rate of substitution is $MRS(x_A, x_B) = -x_B/x_A$. This is the slope of his indifference curve at (x_A, x_B). The slope of Charlie's budget line is $-1/2$ (give a numerical answer).

(f) Write an equation that implies that the budget line is tangent to an indifference curve at (x_A, x_B). $-x_B/x_A = -1/2.$ There are many solutions to this equation. Each of these solutions corresponds to a point on a different indifference curve. Use pencil to draw a line that passes through all of these points.

(g) The best bundle that Charlie can afford must lie somewhere on the line you just penciled in. It must also lie on his budget line. If the point is outside of his budget line, he can't afford it. If the point lies inside of his budget line, he can afford to do better by buying more of both goods. On your graph, label this best affordable bundle with an E. This happens where $x_A =$ 20 and $x_B =$ 10. Verify your answer by solving the two simultaneous equations given by his budget equation and the tangency condition.

(h) What is Charlie's utility if he consumes the bundle $(20, 10)$? 200.

(i) On the graph above, use red ink to draw his indifference curve through (20,10). Does this indifference curve cross Charlie's budget line, just touch it, or never touch it? Just touch it.

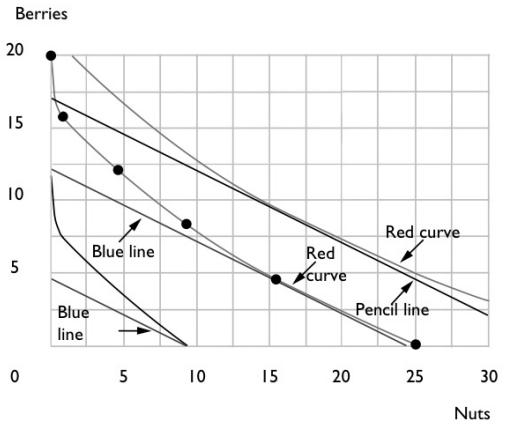
(a) The commodity bundle $(25, 0)$ gives Ambrose a utility of 20. Other points that give him the same utility are $(16, 4)$, (9, 8), (4, 12), (1, 16), and (0, 20). Plot these points on the axes below and draw a red indifference curve through them.

(b) Suppose that the price of a unit of nuts is 1, the price of a unit of berries is 2, and Ambrose's income is 24. Draw Ambrose's budget line with blue ink. How many units of nuts does he choose to buy? 16 units.

(c) How many units of berries? 4 units.

(d) Find some points on the indifference curve that gives him a utility of 25 and sketch this indifference curve (in red).

(e) Now suppose that the prices are as before, but Ambrose's income is 34. Draw his new budget line (with pencil). How many units of nuts will he choose? 16 units. How many units of berries? 9 units.



(f) Now let us explore a case where there is a "boundary solution." Suppose that the price of nuts is still 1 and the price of berries is 2, but Ambrose's income is only 9. Draw his budget line (in blue). Sketch the indifference curve that passes through the point $(9, 0)$. What is the slope of his indifference curve at the point $(9, 0)$? $-2/3$.

(g) What is the slope of his budget line at this point? $-1/2$.

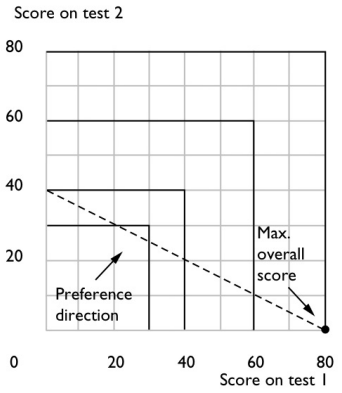
(h) Which is steeper at this point, the budget line or the indifference curve? Indifference curve.

(i) Can Ambrose afford any bundles that he likes better than the point $(9, 0)$? No.

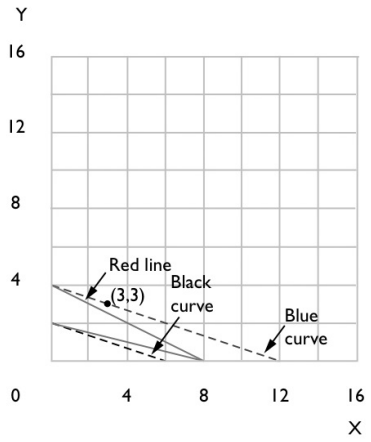
5.5

(b) Given that she spends a total of 400 minutes studying, Nancy will maximize her overall score by achieving a score of 80 on the first examination and 0 on the second examination.

(c) Her overall score for the course will then be 80.



5.7



(b) On the same graph, use red ink to draw Linus's budget line if the price of x is 1 and the price of y is 2 and his income is 8. What bundle does Linus choose in this situation? (0,4).

(c) What bundle would Linus choose if the price of x is 1, the price of y is 4, and his income is 8? (8,0).

1. For each of the following utility functions, find the MRS.

A) $x_1 x_2$. $MRS = -\frac{\frac{\partial(x_1 x_2)}{\partial x_1}}{\frac{\partial(x_1 x_2)}{\partial x_2}} = -\frac{x_2}{x_1}$

B) $(2x_1)^3 x_2$. $MRS = -\frac{\frac{\partial\left((2x_1)^3 x_2\right)}{\partial x_1}}{\frac{\partial((2x_1)^3 x_2)}{\partial x_2}} = -\frac{3x_2}{x_1}$

C) $(2x_1)^2 (2x_2)^2$. $MRS = -\frac{\frac{\partial\left((2x_1)^2(2x_2)^2\right)}{\partial x_1}}{\frac{\partial((2x_1)^2(2x_2)^2)}{\partial x_2}} = -\frac{x_2}{x_1}$

D) $\sqrt{x_1^2 - 3x_2}$. $MRS = -\frac{\frac{\partial\left(\sqrt{x_1^2-3x_2}\right)}{\partial x_1}}{\frac{\partial\left(\sqrt{x_1^2-3x_2}\right)}{\partial x_2}} = \frac{2x_1}{3}$. *Note that the MRS is positive. This is because the preferences are not monotonic. Utility is decreasing in* x_2.

E) $ln\left[\left(2x_1^3\right)\left(2x_2^3\right)\right]$. $MRS = -\frac{\frac{\partial\left(ln\left[\left(2x_1^3\right)\left(2x_2^3\right)\right]\right)}{\partial x_1}}{\frac{\partial\left(ln\left[\left(2x_1^3\right)\left(2x_2^3\right)\right]\right)}{\partial x_2}} = -\frac{x_2}{x_1}$

F) $x_1 + x_1 x_2$. $MRS = -\frac{\frac{\partial(x_1 + x_1 x_2)}{\partial x_1}}{\frac{\partial(x_1 + x_1 x_2)}{\partial x_2}} = -\frac{x_2+1}{x_1}$

Note that A,C,E have the same MRS. They represent the same preferences.

2. Sketch a few indifference curves of the following utility functions.

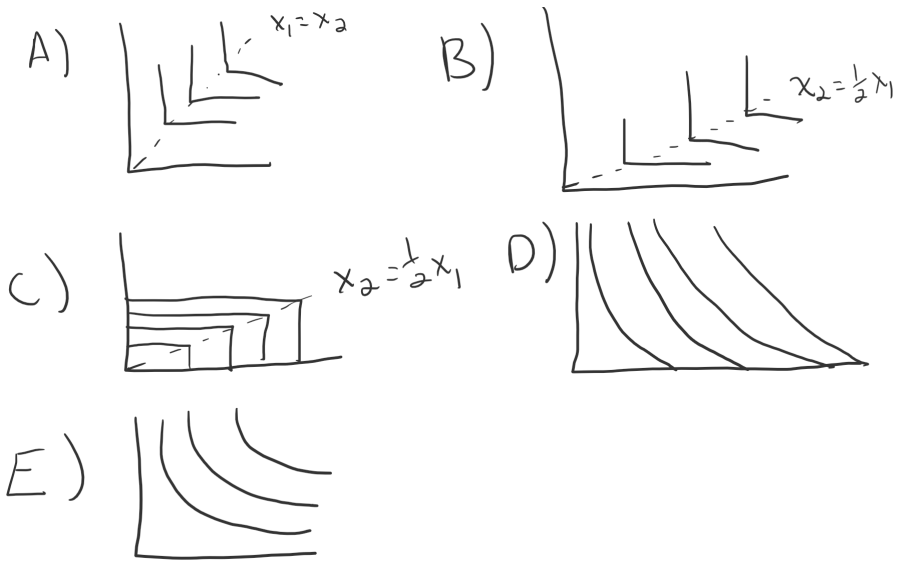
A) $min\{x_1, x_2\}$

B) $min\{x_1, 2x_2\}$

C) $max\{x_1, 2x_2\}$

D) $x_1 + x_1 x_2$

E) $x_1^2 x_2^2$



3. At prices $p_1 = 1$ and $p_2 = 2$ with income $m = 10$, what bundle of goods is optimal for the following utility functions?

A) $x_1 + 2x_2$. **Any bundle such that** $x_1 + 2x_2 = 10$**.**

B) $x_1 x_2$. $(5, 2.5)$

C) $x_1 + ln\,(x_2)$. $\left(9, \frac{1}{2}\right)$

D) $min\,\{x_1, x_2\}$. $\left(\frac{10}{3}, \frac{10}{3}\right)$

E) $min\,\{2x_1, x_2\}$. $(2, 4)$