HW 2 Soln

6.1 (0) Charlie is back—still consuming apples and bananas. His utility function is $U(x_A, x_B) = x_A x_B$. We want to find his demand function for apples, $x_A(p_A, p_B, m)$, and his demand function for bananas, $x_B(p_A, p_B, m)$.

(a) When the prices are p_A and p_B and Charlie's income is m, the equation for Charlie's budget line is $p_A x_A + p_B x_B = m$. The slope of Charlie's indifference curve at the bundle (x_A, x_B) is $-MU_1(x_A, x_B)/MU_2(x_A, x_B) =$ $-x_B/x_A$. The slope of Charlie's budget line is $-p_A/p_B$. Charlie's indifference curve will be tangent to his budget line at the point (x_A, x_B) if the following equation is satisfied: $p_A/p_B = x_B/x_A$.

(b) You now have two equations, the budget equation and the tangency equation, that must be satisfied by the bundle demanded. Solve these two equations for x_A and x_B. Charlie's demand function for apples is $x_A(p_A, p_B, m) = \frac{m}{2p_A}$, and his demand function for bananas is $x_B(p_A, p_B, m) = \frac{m}{2p_B}$.

(c) In general, the demand for both commodities will depend on the price of both commodities and on income. But for Charlie's utility function, the demand function for apples depends only on income and the price of apples. Similarly, the demand for bananas depends only on income and the price of bananas. Charlie always spends the same fraction of his income on bananas. What fraction is this? 1/2.

6.2 (0) Douglas Cornfield's preferences are represented by the utility function $u(x_1, x_2) = x_1^2 x_2^3$. The prices of x_1 and x_2 are p_1 and p_2.

(a) The slope of Cornfield's indifference curve at the point (x_1, x_2) is $-2x_2/3x_1$.

(b) If Cornfield's budget line is tangent to his indifference curve at (x_1, x_2), then $\frac{p_1 x_1}{p_2 x_2} =$ 2/3. (Hint: Look at the equation that equates the slope of his indifference curve with the slope of his budget line.) When he is consuming the best bundle he can afford, what fraction of his income does Douglas spend on x_1? 2/5.

(c) Other members of Doug's family have similar utility functions, but the exponents may be different, or their utilities may be multiplied by a positive constant. If a family member has a utility function $U(x, y) = c x_1^a x_2^b$ where a, b, and c are positive numbers, what fraction of his or her income will that family member spend on x_1? a/(a+b).

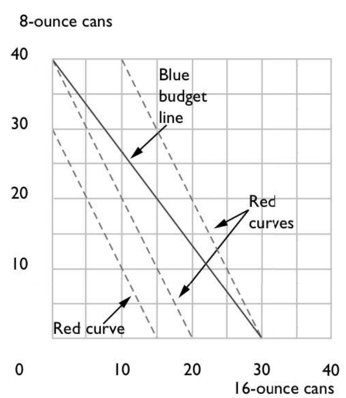
6.3 (0) Our thoughts return to Ambrose and his nuts and berries. Ambrose's utility function is $U(x_1, x_2) = 4\sqrt{x_1} + x_2$, where x_1 is his consumption of nuts and x_2 is his consumption of berries.

(a) Let us find his demand function for nuts. The slope of Ambrose's indifference curve at (x_1, x_2) is $-\frac{2}{\sqrt{x_1}}$. Setting this slope equal to the slope of the budget line, you can solve for x_1 without even using the budget equation. The solution is $x_1 = \left(\frac{2p_2}{p_1}\right)^2$.

(b) Let us find his demand for berries. Now we need the budget equation. In Part (a), you solved for the amount of x_1 that he will demand. The budget equation tells us that $p_1x_1 + p_2x_2 = M$. Plug the solution that you found for x_1 into the budget equation and solve for x_2 as a function of income and prices. The answer is $x_2 = \frac{M}{p_2} - 4\frac{p_2}{p_1}$.

(c) When we visited Ambrose in Chapter 5, we looked at a "boundary solution," where Ambrose consumed only nuts and no berries. In that example, $p_1 = 1$, $p_2 = 2$, and $M = 9$. If you plug these numbers into the formulas we found in Parts (a) and (b), you find $x_1 =$ 16, and $x_2 = -3.5$. Since we get a negative solution for x_2, it must be that the budget line $x_1 + 2x_2 = 9$ is not tangent to an indifference curve when $x_2 \geq 0$. The best that Ambrose can do with this budget is to spend all of his income on nuts. Looking at the formulas, we see that at the prices $p_1 = 1$ and $p_2 = 2$, Ambrose will demand a positive amount of both goods if and only if $M >$ 16.

6.5



(a) At these prices, which size can will she buy, or will she buy some of each? 16-ounce cans.

(b) Suppose that the price of 16-ounce beers remains \$1 and the price of 8-ounce beers falls to \$.55. Will she buy more 8-ounce beers? No.

(c) What if the price of 8-ounce beers falls to \$.40? How many 8-ounce beers will she buy then? 75 cans.

(d) If the price of 16-ounce beers is \$1 each and if Shirley chooses some 8-ounce beers and some 16-ounce beers, what must be the price of 8-ounce beers? \$.50.

(e) Now let us try to describe Shirley's demand function for 16-ounce beers as a function of general prices and income. Let the prices of 8-ounce and 16-ounce beers be p_8 and p_{16}, and let her income be m. If $p_{16} < 2p_8$, then the number of 16-ounce beers she will demand is m/p_{16}. If $p_{16} > 2p_8$, then the number of 16-ounce beers she will demand is 0. If $p_{16} =$ 2 p_8, she will be indifferent between any affordable combinations.

6.7 (1) Mary's utility function is $U(b,c) = b + 100c - c^2$, where b is the number of silver bells in her garden and c is the number of cockle shells. She has 500 square feet in her garden to allocate between silver bells and cockle shells. Silver bells each take up 1 square foot and cockle shells each take up 4 square feet. She gets both kinds of seeds for free.

(a) To maximize her utility, given the size of her garden, Mary should plant 308 silver bells and 48 cockle shells. (Hint: Write down her "budget constraint" for space. Solve the problem as if it were an ordinary demand problem.)

(b) If she suddenly acquires an extra 100 square feet for her garden, how much should she increase her planting of silver bells? `100 extra silver bells.` How much should she increase her planting of cockle shells? `Not at all.`

(c) If Mary had only 144 square feet in her garden, how many cockle shells would she grow? `36.`

(d) If Mary grows both silver bells and cockle shells, then we know that the number of square feet in her garden must be greater than `192.`

8.1 (0) Gentle Charlie, vegetarian that he is, continues to consume apples and bananas. His utility function is $U(x_A, x_B) = x_A x_B$. The price of apples is \$1, the price of bananas is \$2, and Charlie's income is \$40 a day. The price of bananas suddenly falls to \$1.

(a) Before the price change, Charlie consumed 20 apples and 10 bananas per day. On the graph below, use black ink to draw Charlie's original budget line and put the label A on his chosen consumption bundle.

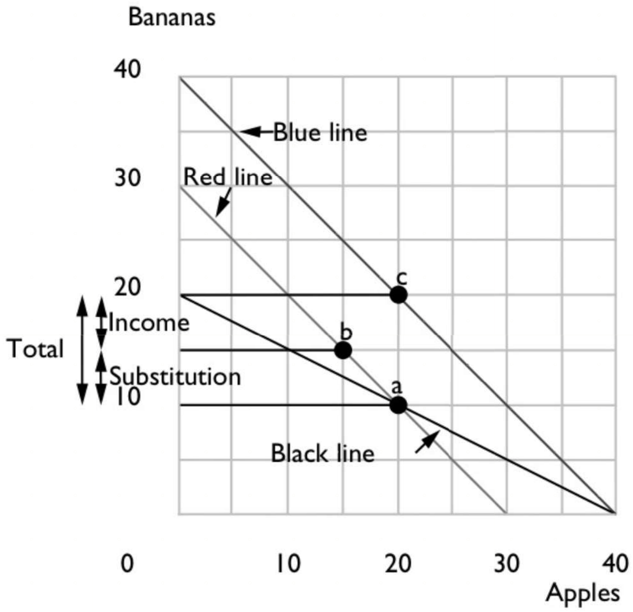
(b) If, after the price change, Charlie's income had changed so that he could exactly afford his old consumption bundle, his new income would have been 30. With this income and the new prices, Charlie would consume 15 apples and 15 bananas. Use red ink to draw the budget line corresponding to this income and these prices. Label the bundle that Charlie would choose at this income and the new prices with the letter B.

(c) Does the substitution effect of the fall in the price of bananas make him buy more bananas or fewer bananas? More bananas. How many more or fewer? 5 more.

(d) After the price change, Charlie actually buys 20 apples and 20 bananas. Use blue ink to draw Charlie's actual budget line after the price change. Put the label C on the bundle that he actually chooses after the price change. Draw 3 horizontal lines on your graph, one from A to the vertical axis, one from B to the vertical axis, and one from C to the vertical axis. Along the vertical axis, label the income effect, the substitution effect, and the total effect on the demand for bananas. Is the

blue line parallel to the red line or the black line that you drew before?

Red line.

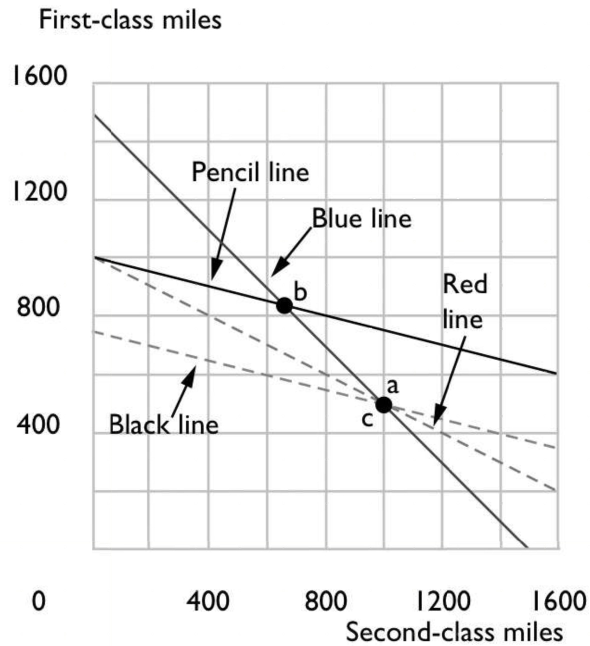


(e) The income effect of the fall in the price of bananas on Charlie's demand for bananas is the same as the effect of an (increase, decrease) increase in his income of $ 10 per day. Does the income effect make him consume more bananas or fewer? More. How many more or how many fewer? 5 more.

(f) Does the substitution effect of the fall in the price of bananas make Charlie consume more *apples* or fewer? Fewer. How many more or fewer? 5 fewer. Does the income effect of the fall in the price of bananas make Charlie consume more apples or fewer? More. What is the total effect of the change in the price of bananas on the demand for apples? Zero.

8.5 (1) Suppose that two goods are perfect complements. If the price of one good changes, what part of the change in demand is due to the substitution effect, and what part is due to the income effect? All income effect.

8.10



(b) Let m_1 be the number of miles she travels by first-class coach and m_2 be the number of miles she travels by second-class coach. Write down two equations that you can solve to find the number of miles she chooses to travel by first-class coach and the number of miles she chooses to travel by second-class coach. $.2m_1 + .1m_2 = 200$, $m_1 + m_2 = 1,500$.

(c) The number of miles that she travels by second-class coach is 1,000.

(d) Just before she was ready to buy her tickets, the price of second-class tickets fell to \$.05 while the price of first-class tickets remained at \$.20. On the graph that you drew above, use pencil to show the combinations of first-class and second-class tickets that she can afford with her \$200 at these prices. On your graph, locate the combination of first-class and second-class tickets that she would now choose. (Remember, she is going to travel as much first-class as she can afford to and still make the 1,500 mile trip on \$200.) Label this point *B*. How many miles does she travel by second class now? 666.66. (Hint: For an exact solution you will have to solve two linear equations in two unknowns.) Is second-class travel a normal good for Agatha? No. Is it a Giffen good for her? Yes.

8.11 (0) We continue with the adventures of Agatha, from the previous problem. Just after the price change from \$.10 per mile to \$.05 per mile for second-class travel, and just before she had bought any tickets, Agatha misplaced her handbag. Although she kept most of her money in her sock, the money she lost was just enough so that at the new prices, she could exactly afford the combination of first- and second-class tickets that she would have purchased at the old prices. How much money did she lose?

\$50. On the graph you started in the previous problem, use black ink to draw the locus of combinations of first- and second-class tickets that she can just afford after discovering her loss. Label the point that she chooses with a C. How many miles will she travel by second class now?

1,000.

(a) Finally, poor Agatha finds her handbag again. How many miles will she travel by second class now (assuming she didn't buy any tickets before she found her lost handbag)? 666.66. When the price of second-class tickets fell from \$.10 to \$.05, how much of a change in Agatha's demand for second-class tickets was due to a substitution effect? None.

How much of a change was due to an income effect? −333.33.

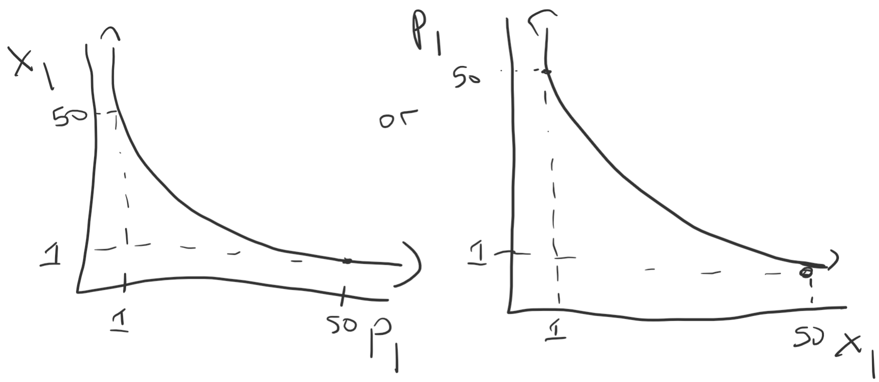
1. A consumer has utility function $x_1 x_2$.

A) What is the consumers demand for x_1 and x_2 as a function of prices and income.

$$x_1 = \frac{\frac{1}{2}m}{p_1}, x_2 = \frac{\frac{1}{2}m}{p_2}$$

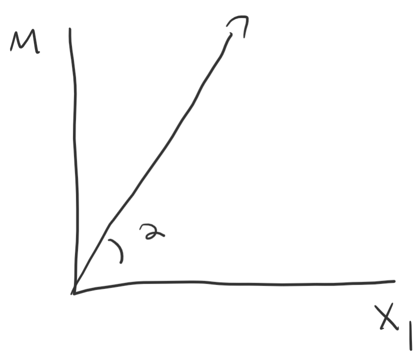
B) Plot the **demand curve for** x_1 when $p_2 = 2$ and $m = 100$.

$$x_1 = \frac{\frac{1}{2}(100)}{p_1} = \frac{50}{p_1}$$



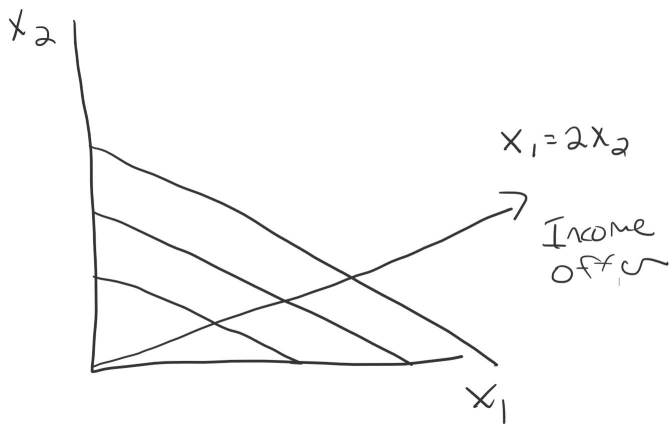
m

C) For $p_1 = 1$ and $p_2 = 2$ sketch the **Engel curve for** x_1.



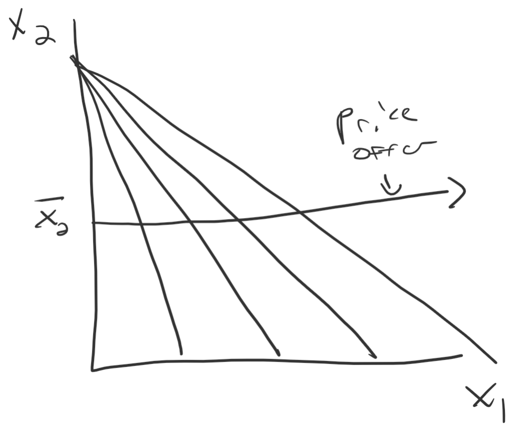
D) For $p_1 = 1$ and $p_2 = 2$ sketch the **Income Offer Curve**.

$$x_1 = \frac{m}{2}, x_2 = \frac{m}{4}$$



E) For $m = 100$ sketch the **price offer curve** for p_1.

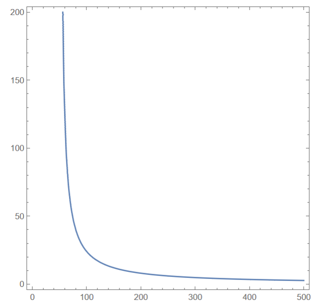
Note here that x_2 does not depend on the price of p_1. So the price offer curve when we hold p_2 fixed will involves bundles where x_2 is fixed at some level.



2.

A. $x_1 = 2\frac{m}{2p_1+p_2}, x_2 = \frac{m}{2p_1+p_2}$

B. $p_1 = \frac{1200}{x_1} - 50$



C. $m = 150x_1$. Plot is a line with slope of 150 going through the origin.

D. A line with slope $\frac{1}{2}$ going through the origin.

E. A line with slope of $\frac{1}{2}$ going through the origin.

3. A consumer has utility function $u(x_1, x_2) = 2x_1 + x_2$.

A. $(12, 0)$

B. $(0, 12)$

C. $m = 3600$

D. $(0, 36)$

E. 12

F. 0

4. A. $(8, 4)$

B. $(6, 3)$

C. 1600

D. $(8, 4)$

E. 0

F. 2