Problem on page 1.

A) $p_1x_1 + p_2x_2 = m$

B) $-\frac{p_1}{p_2}$

C) $-\frac{3}{2}$

D) $-\frac{x_2}{x_1} = -\frac{p_1}{p_1}$

E) $x_1 = \frac{\frac{1}{2}m}{p_1}$

F) $x_1 = \frac{\frac{1}{2}10}{2} = 2.5$

G) Demand goes up. This is a normal good.

H) Nothing happens. They are neither complements nor substitutes.

I) Nothing changes. This is a monotonic transformation of the other utility function and represents the *exact* same preferences.

Problem on page 5.

A) In the short run since $t = 1$ we have the production function:

$$h = s^{\frac{1}{3}}$$

The amount of studio he needs to produce one track is:

$$1 = s^{\frac{1}{3}}$$

$$s = 1$$

The cost of this is 1000.

B) $s^{\frac{1}{3}}t^{\frac{1}{3}}$. The marginal returns for studio time:

$$MP_s = \frac{\partial s^{\frac{1}{3}}t^{\frac{1}{3}}}{\partial s} \quad = \quad \frac{1}{3}s^{-\frac{2}{3}}t^{\frac{1}{3}}$$

This is decreasing if its derivative is negative:

$$\frac{\partial \left(\frac{1}{3}s^{-\frac{2}{3}}t^{\frac{1}{3}}\right)}{\partial s} = \frac{1}{3}\left(-\frac{2}{3}\right)s^{-\frac{5}{3}}t^{\frac{1}{3}} = -\frac{2}{9}s^{-\frac{5}{3}}t^{\frac{1}{3}} < 0$$

We also see that this is a Cobb Douglass production function with exponents less than one and so it has decreasing marginal product.

C) $s^{\frac{1}{3}}t^{\frac{1}{3}}$ has decreasing returns to scale when for any positive z:

$$(zs)^{\frac{1}{3}}\,(zt)^{\frac{1}{3}} < zs^{\frac{1}{3}}t^{\frac{1}{3}}$$

$$z^{\frac{2}{3}}s^{\frac{1}{3}}t^{\frac{1}{3}} < zs^{\frac{1}{3}}t^{\frac{1}{3}}$$

$$z^{\frac{2}{3}} < z$$

This is true for any positive z. Alternatively this is a Cobb Douglass production function with exponents that sum to less than one so it has decreasing returns to scale.

D) To produce h tracks in the short-run where $t = 1$, Kanye needs:

$$h = s^{\frac{1}{3}}$$

$$s = h^3$$

Thus, the cost of producing h tracks is: $c(h) = 1000h^3$
E) The marginal cost is:

$$MC = \frac{\partial c(h)}{\partial h} = \frac{\partial \left(1000h^3\right)}{\partial h} = 3000h^2$$

This is increasing the derivative of marginal cost is positive:

$$\frac{\partial MC(h)}{\partial h} = \frac{\partial \left(3000h^2\right)}{\partial h} = 6000h > 0$$

F) His profit function is $\pi(h) = 12000h - 1000h^3$. To maximize this he finds the point where its derivative is zero:

$$\frac{\partial \pi(h)}{\partial h} = \frac{\partial \left(12000h - 1000h^3\right)}{\partial h} = 0$$

$$12000 = 3000h^2$$

$$4 = h^2$$

$$2 = h$$

He chooses to produce 2 tracks. He uses: $s = (2)^3 = 8$ days of studio time and earns a profit of: $12000(2) - 1000(2)^3 = 24000 - 8000 = 16000$.
G) We know he can earn 16000 if he does not take this offer. How much can he earn if he does? We first need to find his cost function for producing tracks if he does take the offer.

$$h = s^{\frac{1}{3}} (1000)^{\frac{1}{3}} = 10s^{\frac{1}{3}}$$

He needs $10s^{\frac{1}{3}} = h$ or $s = \frac{1}{1000}h^3$ days of studio time to produce h tracks. This costs him (remember to include the cost of talent):

$$c(h) = \frac{1000}{1000}h^3 + 1000000 = h^3 + 100000$$

Let's maximize profit given this cost function:

$$\pi = 12000h - h^3 - 100000$$

This is maximized where:

$$\frac{\partial \left(12000h - h^3 - 100000\right)}{\partial h} = 12000 - 3h^2$$

$$\sqrt{4000} = h$$

His profit is about 405000. Much larger than what he can get if he does not take the offer.

Problem on page 6 (selected solutions)

B) $1000 - p = 200$. $p^* = 800$.

C) $1000-(p+100) = 200$. $p^* = 700$. This is the price suppliers get. Consumers pay 800.

D) $1000 - p = p$. $p = 500$.

E) $1000 - (p+100) = p$. $p = 450$. This is the price suppliers get. Consumers pay 550.

F) 2500

Question from page 7 of the practice problems.

A) If he only consumes this month, he can borrow up to $\left(\frac{1}{1+r}\right) m_2$. This if he only consumes this month he can consume $m_1 + \left(\frac{1}{1+r}\right) m_2$. Likewise, if he only consumes next month, he saves all m_1 and gets back $(1+r)\, m_1$ next month. This means he can consume $(1+r)\, m_1 + m_2$ next month if he only consumes next month. Plugging in $m_1 = 4000$ and $m_2 = 5000$ we get $4000 + \left(\frac{1}{1+r}\right) 5000$ and $(1+r)\, 4000 + 5000$.

B) The intercepts are the points given above. The horizontal intercept is $4000 + \left(\frac{1}{1+r}\right) 5000$ and the vertical intercept is $(1+r)\, 4000 + 5000$. The slope of the line is $-(1+r)$.

C) The MRS is $-\frac{c_2}{c_1}$ setting this equal to the slope of the budget equation we get:

$$c_2 = (1+r)\, c_1$$

Plugging this back into the budget equation $(1+r)\, c_1 + c_2 = (1+r)\, 4000 + 5000$ we get:

$$c_1 = \frac{4000 + \left(\frac{1}{1+r}\right) 5000}{2}$$

$$c_2 = \frac{(1+r)\, 4000 + 5000}{2}$$

D) If he does not borrow or save, $c_1 = m_1$ and $c_2 = m_2$ using either of the equations above, we can solve for the r that makes this happen:

$$5000 = \frac{(1+r)\, 4000 + 5000}{2}$$

$$r = \frac{1}{4}$$

E) [This one is a little tricky]. We know this consumer cannot become a borrower. This is because, if he were to become a borrow, he would be choosing something that was *strictly affordable* to him when he chose to neither borrow nor save. He is at least as well off as he was before. Unlike in the usual case where a consumer is for instance a lender when the interest rate increases, no bundle becomes available that is obviously strictly better than the one that he was consuming. However, we know he can't be worse off because he could always just continue consuming the same point he was before.

F) Inflation was not covered in class. Don't worry about this part.

Problem on page 8

A) $\frac{m_i}{2p}$. Elasticity is $-\frac{\partial \frac{m_i}{2p}}{\partial p}\frac{p}{\frac{m_i}{2p}} = 1$. This is unit elastic demand. This does not depend on the price.

B) The market demand is the sum of all ten consumer's demands:

$$\sum_{i=1}^{10} \frac{m_i}{2p} = \frac{\sum_{i=1}^{10} m_i}{2p}$$

Since everyone has an income of 20 the market demand is:

$$\frac{\sum_{i=1}^{10} 20}{2p} = \frac{100}{p}$$

C) Elaxitity is:

$$\varepsilon_p = -\frac{\partial \frac{100}{p}}{\partial p}\frac{p}{\frac{100}{p}} = -\left(-\frac{100}{p^2}\right)\frac{p}{\frac{100}{p}} = \frac{100}{p^2}\frac{p}{100}p$$

$$= \frac{100}{p^2}\frac{p^2}{100} = 1$$

This implies that as the price goes up by one-percent, demand will decrease by one-percent.

D) Market demand is now:

$$\sum_{i=1}^{20} \frac{m_i}{2p} = 10\frac{20}{2p} + 10\frac{20}{p} = \frac{300}{p}$$

E) The new consumers also have unit-elastic demand:

$$-\frac{\partial \frac{20}{p}}{\partial p}\frac{p}{\frac{20}{p}} = -\left(-\frac{20}{p^2}\right)\frac{p}{\frac{20}{p}} = 1$$

$$-\left(-\frac{20}{p^2}\right)\frac{p}{\frac{20}{p}} = \left(\frac{20}{p^2}\right)\frac{p}{20}p = \frac{20}{p^2}\frac{p^2}{20} = 1$$

Market demand is also unit-elastic (by a similar calculation).

$$-\frac{\partial \frac{300}{p}}{\partial p}\frac{p}{\frac{300}{p}} = 1$$

F) The first consumer type has demand $\frac{m_i}{2p}$. Their income elasticity of demand is:

$$\frac{\partial \frac{m_i}{2p}}{\partial m} \frac{m_i}{\frac{m_i}{2p}} = \frac{1}{2p} 2p = 1$$

Second consumer type's demand is:

$$\frac{\partial \frac{m_i}{p}}{\partial m} \frac{m_i}{\frac{m_i}{p}} = \frac{1}{p} p = 1$$

The market demand is also unit-elastic with respect to income. We can say that a 1% in incomes will lead to a 1% increase in demand.

Problem on page 12.

A) The budget is a line with slope of -2 passing from the point $(0, 20)$ to the point $(10, 0)$. The indifference curves are L-shaped curves with kinks only the 45-degree line.

B) We need $x_1 = x_2$ to be consuming where the indifference curve just touches the budget line. Plugging this into the budget equation, we get:

$$x_1 = x_2 = \frac{20}{3}$$

C) Now we need a point on the budget line where $2x_1 = x_2$. Plugging this into the budget equation, we get

$$x_1 = 5, x_2 = 10$$

D) The budget line has a new intercept. The consumer could buy 30 eggs if they only bought eggs. Thus the budget line starts at $(30, 0)$ then decreases with slope -4 to the point $(10, 5)$ then returns to normal with a slope of -2.

E) Since $(5, 10)$ is still on this budget equation and there is no better point on the budget equation where $2x_1 = x_2$, this is still optimal.

Question on page 13.

Technology 1 costs $l^2 + 10l + 100$. Technology 2 costs $l^2 + 1000$

A) Technology 1: $ATC = \frac{l^2+10l+100}{l} = l + 10 + \frac{100}{l}$. $AVC = l + 10$

Technology 2: $ATC = l + \frac{1000}{l}$. $AVC = l$

B) Technology 1: $MC = \frac{\partial\left(l^2+10l+100\right)}{\partial l} = 2l + 10$

Technology 2: $MC = \frac{\partial\left(l^2+1000\right)}{\partial l} = 2l$

Both have increasing marginal costs.

C) Decreasing returns to scale.

D) $AVC = l + 10$ is increasing in l and so it has a minimum at $l = 0$ where AVC is 10. The place where $MC = AVC$ is $2l + 10 = l + 10$ or $l = 0$.

E) We just calculate which technology costs least for $l = 50$.

$$C_1\left(50\right) = 50^2 + 50 * 10 + 100 = 3100$$

$$C_2\left(50\right) = 50^2 + 1000 = 3500$$

The firm uses technology 1.

Question on page 18.

A) This is a line with slope -1 passing through the points $(10,0)$ and $(0,10)$.

B) The new line is one with slope of -2 passing through the points $(5,0)$ and $(0,10)$.

C) The new budget starts at $(0,10)$ and decreases with a slope of $-\frac{1}{2}$ to the point $(4,8)$ then decreases with slope -1 until it gets to $(12,0)$.

D) Greg must buy where $x_1 = x_2$ to be doing something optimal. He can buy the bundle $(4,4)$ at the total cost of \$6 and still have \$4 left over to buy an additional 2 of each good and arrive at the bundle of $(6,6)$.

Problem on page 24.

A) *Decreasing.*

$$MP = \frac{\partial\left(x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}\right)}{\partial x_1} \quad = \quad \frac{\sqrt{x_2}}{2\sqrt{x_1}}$$

$$\frac{\partial MP}{\partial x_1} = \frac{\partial\left(\frac{\sqrt{x_2}}{2\sqrt{x_1}}\right)}{\partial x_1} = -\frac{\sqrt{x_2}}{4x_1^{3/2}} < 0$$

B)

$$TRS = -\frac{x_2}{x_1}$$

C)

$$x_2^* = \frac{y}{2}$$

$$x_1^* = 2y$$

D)

$$c(y) = 20y$$

E) *Constant.*

$$MC = \frac{\partial(20y)}{\partial y} = 20$$

Question on page 19

$$u = x_1 x_2$$

A) $p_1 x_1 + p_2 x_2 = m$. $-\frac{p_1}{p_2}$

B) $-\frac{x_2}{x_1}$

C) $-\frac{3}{4}$

D) $-\frac{x_2}{x_1} = -\frac{p_1}{p_2}$

E) $x_1 = \frac{\frac{1}{2}m}{p_1}$

F) $\frac{\frac{1}{2}10}{1} = 5$

G) Demand goes up. Grapes are a normal good.

H) Nothing happens. They are neither substitutes nor complements.

I) This is a monotonic transformation of the old utility function. Nothing about his consumption changes.

Question on Page 23.

$$q_d = 300 - p$$

$$q_s = \frac{1}{2}p$$

A) We find where $q_d = q_s$. $300 - p = \frac{1}{2}p$. Or $p = 200$. At this price, the quantity sold is $q^* = 100$.

B) The equilibrium price and quantity both increase.

C) We find where $q_d = q_s$. $300 - (p_s + 60) = \frac{1}{2}p_s$

$$160 = p_s$$

Suppliers receive 160. Consumers pay $160 + 60 = 220$. At this price, $q = 80$ are sold.

D) DWL is $\frac{60*20}{2} = 600$

Burden: Consumers pay $20 more than they were before.

Suppliers get $40 less than they were before.

Problem on page 25

A) This month he can consume up to $200+\frac{1}{1+\frac{1}{2}}600=600$. Next month he can consume up to $\left(1+\frac{1}{2}\right)200+600=900$.

B) The line goes through these two intercepts $(600,0)$ and $(0,900)$ and has slope $-\left(1+\frac{1}{2}\right)=-\frac{3}{2}$.

C) The tangency condition of $-\frac{c_2}{c_1}=-\frac{3}{2}$. Plugging this into the budget equation

$$\frac{3}{2}c_1+c_2=\left(1+\frac{1}{2}\right)200+600$$

Or more simply

$$\frac{3}{2}c_1+c_2=900$$

We get:

$$c_1=300, c_2=450$$

Since $c_1>m_1$, he is a borrower.

D) Since he is a borrower, if the interest rate goes down he remains a borrower and is strictly better off.

E) For a generic interest rate, the tangency condition is

$$-\frac{c_2}{c_1}=-\left(1+r\right)$$

Or

$$c_2=c_1\left(1+r\right)$$

If he neither borrows nor saves then $c_1=200$ and $c_2=600$. Let's plug these in:

$$600=200\left(1+r\right)$$

$$r=2$$

Problem on page 30.

A)

$$atc_1(r) = \frac{c_1(r)}{r} = \frac{r^2}{r} = r$$

$$atc_2(r) = \frac{\frac{1}{2}r^2 + 10r}{r} = \frac{1}{2}r + 10$$

B) Both of these have increasing marginal costs.

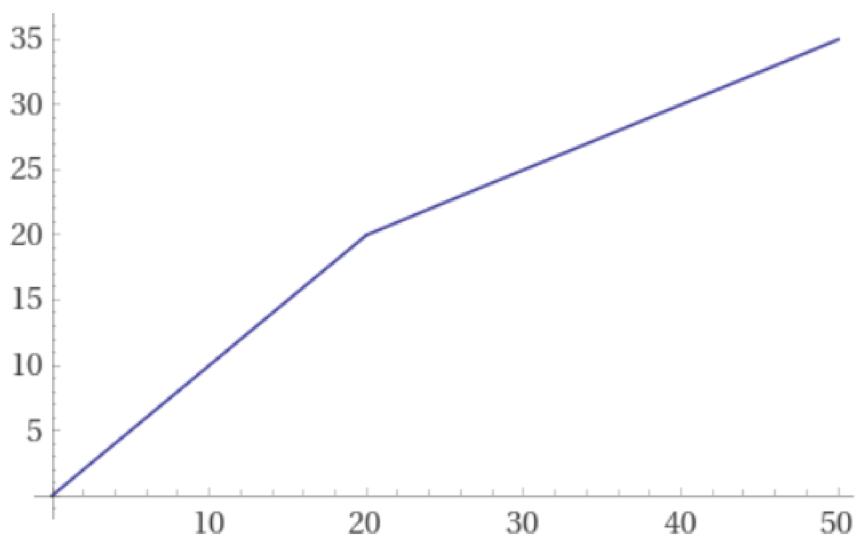
$$mc_1(r) = \frac{\partial\left(r^2\right)}{\partial r} = 2r$$

$$\frac{\partial mc_1(r)}{\partial r} = \frac{\partial(2r)}{\partial r} = 2 > 0$$

$$mc_2(r) = \frac{\partial\left(\frac{1}{2}r^2 + 10r\right)}{\partial r} = r + 10$$

$$\frac{\partial mc_2(r)}{\partial r} = \frac{\partial(r + 10)}{\partial r} = 1 > 0$$

C) In the long run, the firm will choose the cheaper technology:



D) h from 0 to 20

E) If they are a price taker, they set $p = mc(h)$. Using technology 1 the firm wants to produce 10 units.

$$p = 2r$$

$$10 = r^*$$

For technology 2:

$$r + 10 = 20$$

$$10 = r^*$$

They get the same revenue regardless of which technology they choose, but at $r = 10$ technology 1 is cheaper. So, they choose technology 1.

Question on page 32:

A) $Q = 48 - 4p$

B) $-\frac{\partial(48-4p)}{\partial p}\frac{p}{48-4p} = \frac{4p}{48-4p}$. At $p = 4$. $\frac{16}{48-16} = \frac{1}{2}$. Demand is inelastic.

C) $\pi = q\left(12 - \frac{1}{4}q\right) - 2q$. This is maximized at $\frac{\partial\left(q\left(12-\frac{1}{4}q\right)-2q\right)}{\partial q} = 10 - \frac{q}{2} = 0$ or $q = 20$. At this quantity, $p = 7$. Profit is $20 * 7 - 2 * 20 = 100$

D) Inverse demand for Greg: $p = 24 - q$. Inverse demand for Taylor: $p = 8 - \frac{1}{3}q$.

Profit function for Greg: $\pi = q(24 - q) - 2q$. This is maximized at $\frac{\partial(q(24-q)-2q)}{\partial q} = 22 - 2q$ or $q = 11$ and at this quantity, Greg will pay $p = 13$. Profit is $11 * 13 - 2 * 11 = 121$.

Profit function for Taylor: $\pi = q\left(8 - \frac{1}{3}q\right) - 2q$. This is maximized at $\frac{\partial\left(q\left(8-\frac{1}{3}q\right)-2q\right)}{\partial q} = 6 - \frac{2q}{3}$ or $q = 9$ and at this quantity, Taylor will pay $p = 5$. Profit is $9 * 5 - 2 * 9 = 27$.

Total profit is $121 + 27 = 148$. \$48 more profit than setting one price for both.