1 Solving Consumer Utility Maximization

Under the assumption of monotonic preferences.

At the optimal bundle, the indifference curve through that bundle **cannot pass through the budget line.**

The optimal bundle must be on the budget line and on an indifference curve that just touches but does not cross through the budget line.

Three possibilities for an optimal bundle.

1. "Interior" Tangent

If you can take derivatives of the utility function, the a necessary condition for bundle that is optimal and contains some of both goods is:

$$MRS = -\frac{p_1}{p_2}$$

$$-\frac{MU_1}{MU_2} = -\frac{p_1}{p_2}$$

$$\frac{MU_1}{p_1} = \frac{MU_2}{p_2}$$

Marginal utility divided by price is roughly the extra utility you get by spending one dollar more on a good.

Budget: $x_1 + 2x_2 = 12$. Utility: $u(x_1, x_2) = x_1 x_2$

$$x_1 = 12 - 2x_2$$

$$-\frac{\frac{\partial(x_1 x_2)}{\partial x_1}}{\frac{\partial(x_1 x_2)}{\partial x_2}} = -\frac{p_1}{p_2}$$

$$\frac{x_2}{x_1} = \frac{p_1}{p_2}$$

$$x_2 p_2 = x_1 p_1$$

Plugging in the prices we get the **equal slope condition:**

$$2x_2 = x_1$$

Budget condition:

$$x_1 + 2x_2 = 12$$

Solving these conditions simultaneously:

$$2x_2 + 2x_2 = 12$$

$$4x_2 = 12$$

$$x_2 = 3, x_1 = 6$$

$$(6, 3)$$

At the optimal bundle, the amount of money spend on both goods is the same.

Another smooth utility example.

$$x_1 + 2x_2 = 12, u\left(x_1, x_2\right) = x_1^2 x_2$$

$$-\frac{\frac{\partial\left(x_1^2 x_2\right)}{\partial x_1}}{\frac{\partial\left(x_1^2 x_2\right)}{\partial x_2}} = -\frac{1}{2}$$

$$-\frac{2x_1 x_2}{x_1^2} = -\frac{1}{2}$$

$$-\frac{2x_2}{\frac{x_1 x_1}{x_1}} = -\frac{1}{2}$$

$$\frac{2x_2}{x_1} = \frac{1}{2}$$

Equal slope condition:

$$4x_2 = x_1$$

Budget constraint:

$$x_1 + 2x_2 = 12$$

$$6x_2 = 12$$

$$x_1 = 8, x_2 = 2$$

$u\left(x_1, x_2\right) = x_1^{\alpha} x_2^{\beta}, p_1 x_1 + p_2 x_2 = m.$

$$-\frac{\frac{\partial\left(x_1^{\alpha} x_2^{\beta}\right)}{\partial x_1}}{\frac{\partial\left(x_1^{\alpha} x_2^{\beta}\right)}{\partial x_2}} = -\frac{p_1}{p_2}$$

$$Solve[\{-\frac{\alpha * x_2}{\beta * x_1} == -\frac{p_1}{p_2}, p_1 * x_1 + p_2 * x_2 == m\}, \{x_1, x_2\}]$$

$$x_1 = \frac{\frac{\alpha}{\alpha+\beta} m}{p_1}, x_2 = \frac{\frac{\beta}{\alpha+\beta} m}{p_2}$$

$$x_1 + x_2 = 100, u\left(x_1, x_2\right) = \left(ln\left(x_1 x_2\right) + 10\right)^2$$

2. "Interior" Touching but not tangent

Budget: $x_1 + 2x_2 = 12$. Utility: $u\left(x_1, x_2\right) = min\left\{x_1, x_2\right\}$

What replaces the equal slope condition in a perfect complements problem is "no waste condition"

$$x_1 = x_2$$

$$x_1 + 2x_2 = 12$$

$$x_2 + 2x_2 = 12$$

$$x_1 = 4, x_2 = 4$$

Budget: $x_1 + 2x_2 = 12$. Utility: $u(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$

$$\frac{1}{2}x_1 = x_2$$

$$x_1 + 2x_2 = 12$$

$$x_1 + 2\left(\frac{1}{2}x_1\right) = 12$$

$$x_1 = 6, x_2 = 3$$

$$(6, 3)$$

3. Corner

Budget: $x_1 + 2x_2 = 12$. Utility: $u(x_1, x_2) = x_1 + x_2$

$$-\frac{1}{1} = -\frac{1}{2}$$

at least one of the two endpoints has to optimal. $\left(\frac{m}{p_1}, 0\right), \left(0, \frac{m}{p_2}\right)$

$$(12, 0), (0, 6)$$

$$u(12, 0) = 6$$

$$u(0, 6) = 12$$

Budget: $x_1 + 2x_2 = 12$. Utility: $u(x_1, x_2) = x_1 + 2x_2$

$$(12, 0), (0, 6)$$

$$u(12, 0) = 12, u(0, 6) = 12$$

This consumer is willing to buy anything on the budget line $x_1 + 2x_2 = 12$.

One last example. Quasi-linear utility. $u(x_1, x_2) = ln(x_1) + x_2$, $p_1 = 1, p_2 = 1, m = 10$

Budget line:

$$x_1 + x_2 = 10$$

Equal slope condition:

$$MRS = \frac{\frac{\partial(\log(x_1)+x_2)}{\partial x_1}}{\frac{\partial(\log(x_1)+x_2)}{\partial x_2}} = -\frac{\frac{1}{x_1}}{1} = -\frac{1}{x_1}$$

$$-MRS = -R$$

$$-\frac{1}{x_1} = -\frac{1}{1}$$

$$x_1 = 1$$

$$x_2 = 10 - 1$$

$$(1, 9)$$

2 Demand

Marshallian Demand

Changes in Income

Normal/Inferior

Income Offer Curve

Engel Curve

Changes in Own Price

Ordinary/Giffen

Plotting Demand

Price Offer Curve