1 Demand

1.1 Marshallian Demand

The outcome of utility maximization is an amount of each good demanded. We call these amounts the "Marshallian Demand".

$$x_1\left(p_1, p_2, m\right), x_2\left(p_1, p_2, m\right)$$

1.1.1 Example of Perfect Complements

$u = min\left\{x_1, x_2\right\}$.

$$x_1 = x_2$$

$$p_1 x_1 + p_2 x_2 = m$$

$$x_1 = \frac{m}{p_1 + p_2}, x_2 = \frac{m}{p_1 + p_2}$$

1.1.2 Example of Cobb Douglass

$u = x_1 x_2$

$$\frac{\alpha}{\alpha + \beta}, \frac{\beta}{\alpha + \beta}$$

$$x_1 = \frac{\left(\frac{1}{2}\right) m}{p_1}, x_2 = \frac{\left(\frac{1}{2}\right) m}{p_2}$$

1.2 Changes in Income

What happens to demand when we hold p_1, p_2 fixed and change m?

1.2.1 Normal/Inferior Goods

A good is called **normal** if demand goes **up** when income goes up.
A good is called **inferior** if demand goes **down** when income goes up
Top Ramen, Public Transportation, Potatoes, Small Cars

Good's don't always have to just be one thing. They can be normal for low income ranges and become inferior for high income range.

1.2.2 A good is never "always inferior".

To be inferior, demand has to decrease when income increases. To decrease it has have increased at some point.

At zero income, demand has to be zero.

1.2.3 Examples:

$$x_1 = \frac{m}{p_1 + p_2}$$

$$\frac{\partial \left(\frac{m}{p_1+p_2} \right)}{\partial m} = \frac{1}{p_1 + p_2} > 0$$

Since as m increases, x_1 always increases, x_1 is normal.

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

$$\frac{\partial \left(\frac{\frac{1}{2}m}{p_1} \right)}{\partial m} = \frac{\frac{1}{2}}{p_1} = \frac{1}{2p_1} > 0$$

1.2.4 Plots: "Engel Curve"

Engle curve plots the relationship between income and a good with **income on the vertical axis** holding prices fixed.

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

To the get plot for the engle curve, isolate m from the demand function.

$$2p_1 x_1 = m$$

This is a line with slope $2p_1$. Let's plot the engel curve where $p_1 = 1$. $m = 2x_1$

At $p_1 = 2$ the engel curve is $4x_1 = m$

For perfect complements:

$$x_1 = \frac{m}{p_1 + p_2}$$

$$m = (p_1 + p_2)\, x_1$$

Suppose $p_1 = p_2 = 1$

$$m = 2x_1$$

1.2.5 Plots: "Income Offer Curve"

This is a plot showing **bundles** that are consumed as income changes.

Suppose $p_1 = p_2 = 1$ and $u = x_1 x_2$ then demand is:

$$x_1 = \frac{1}{2}m, x_2 = \frac{1}{2}m$$

At $m = 2$, $(1, 1)$. At $m = 4$, $(2, 2)$, at $m = 6$ $(3, 3)$

To get the income offer curve, eliminate m from the system of equations defined by the demands.

$$x_1 = \frac{1}{2}m, x_2 = \frac{1}{2}m$$

Solve for m in one of the equations:

$$2x_1 = m$$

Plug this into the other demand:

$$x_2 = \frac{1}{2}(2x_1) = x_1$$

$$x_2 = x_1$$

For perfect complements $u = min\{x_1, x_2\}$ with $p_1 = 1, p_2 = 2$

$$x_1 = \frac{m}{3}, x_2 = \frac{m}{3}$$

$$3x_1 = m$$

$$x_2 = \frac{3x_1}{3} = x_1$$

$$x_2 = x_1$$

Suppose we had $u = min\left\{\frac{1}{2}x_1, x_2\right\}$, $p_1 = 1, p_2 = 2$

$$x_2 = \frac{1}{2}x_1$$

$$1x_1 + 2x_2 = m$$

$$x_1 + 2\left(\frac{1}{2}x_1\right) = m$$

$$2x_1 = m, x_1 = \frac{m}{2}$$

$$x_1 = \frac{m}{2}, x_2 = \frac{m}{4}$$

Find the income offer curve:

$$2x_1 = m$$

Plug this into the other demand:

$$x_2 = \frac{2x_1}{4} = \frac{1}{2}x_1$$

$$x_2 = \frac{1}{2}x_1$$

1.3 Changes in "Own" Price

Holding income and the price of the other good fixed, how does demand depend on the "own" price.

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

$$x_1 = \frac{m}{p_1 + p_2}$$