# 1 Slutsky Decomposition

## 1.1 Two Effects

**Substitution Effect.** When the price of a good goes up, the consumer will substitute away from that good to buy similar goods that are now relatively cheaper.
**Income Effect.** When the price of a good goes up, the same income can buy less and this will lead to a change in demand depending on whether the good is normal or inferior.

**Total Effect** is the Sum of the Income Effect and the Substitution Effect.

## 1.2 The Thought Experiment

The thought experiment involved in the slutsky decomposition allows us to **determine the substitution effect.**

1. Find demand at the old prices.
2. Find demand at the new prices

The difference is the **total effect.**

3. Calculate how much income the consumer would need to buy the old bundle (from part 1) at the new prices. **Compensated Income.**

4. Construct a new budget that uses the new prices and the compensated income.

5. Find what the consumer would choose on the compensated budget line.

6. The difference between part 1 and part 5 can't be due to income. This is the **substitution effect.**

## 1.3 Law of Demand

**The substitution effect is always weakly negative.**

When the price of a good goes up, I will always buy less of the good due to the substitution effect.

When the price of a good goes up:

**Both effects are negative. (Ordinary,Normal)**

*This implies that every normal good is also ordinary.*

**Substitution is negative, income is positive. (Inferior,Ordinary)**

but the positive income effect does not overwhelm the negative substitution effect.

**Substitution is negative, income is positive. (Inferior,Giffen)**

The income effect is so large (and positive) that it overwhelms and negative substitution effect.

## 1.4 Example Problem (Cobb Douglass)

$u = x_1 x_2$. Suppose $p_1 = 1$, $p_2 = 2$, $m = 40$. $p_1$ changes to 2. Find the substitution and income effect on $x_1$.

$$x_1 = \frac{\frac{1}{2}m}{p_1}, x_2 = \frac{\frac{1}{2}m}{p_2}$$

Find the demand under the original prices.

$$x_1 = \frac{\frac{1}{2}(40)}{1}, x_2 = \frac{\frac{1}{2}(40)}{2}$$

$$x_1 = 20, x_2 = 10$$

Find the demand under the new prices.

$$x_1 = \frac{\frac{1}{2}(40)}{2}, x_2 = \frac{\frac{1}{2}(40)}{2}$$

$$x_1 = 10, x_2 = 10$$

The total effect of an increase in $p_1$ from 1 to 2 is a decrease in demand for $x_1$ of 10 units.

To find what part of this 10 unit change is due to substitution, we need to construct the compensated budget line "thought experiment budget".

Find the compensated income. How much money I need to afford the old bundle $(20, 10)$ at new prices $p_1 = 2, p_2 = 2$. The compensated income is $\tilde{m} = 20 * 2 + 10 * 2 = 60$.

What does the consumer buy on the compensated budget.

$$2x_1 + 2x_2 = 60$$

What would the consumer choose with this budget?

$$x_1 = \frac{\frac{1}{2}(60)}{2}, x_2 = \frac{\frac{1}{2}(60)}{2}$$

$$x_1 = 15, x_2 = 15$$

Original demand 20 demand under compensated income at new prices is 15.

The total effect of an increase in $p_1$ from 1 to 2 is a decrease in demand for $x_1$ of 10 units. 5 of this decrease is due to the substitution effect and 5 is due to the income effect.

$$TE = x_1^{original} - x_1^{newprices}$$

$$SE = x_1^{original} - x_1^{compensated}$$

$$IE = x_1^{compensated} - x_1^{newprices}$$

### 1.5 Example Problem (Perfect Complements)

Suppose $u = min\{x_1, x_2\}$. Suppose $p_1 = 1$, $p_2 = 2$, $m = 40$. $p_1$ changes to 2. Find the substitution and income effect.

$$x_1 = x_2$$

$$p_1x_1 + p_2x_2 = m$$

$$p_1x_1 + p_2x_1 = m$$

$$p_1 + p_2(x_1) = m$$

$$x_1 = \frac{m}{p_1 + p_2}, x_2 = \frac{m}{p_1 + p_2}$$

Demand under old prices:

$$x_1 = \frac{40}{1+2}, x_2 = \frac{40}{1+2}$$

$$x_1 = \frac{40}{3}, x_2 = \frac{40}{3}$$

Demand under new prices:

$$x_1 = \frac{40}{2+2}, x_2 = \frac{40}{2+2}$$

$$x_1 = 10, x_2 = 10$$

Total effect on $x_1$:

$$\frac{40}{3} - 10 = \frac{10}{3}$$

$$\frac{40}{3} - \frac{30}{3} = \frac{10}{3}$$

Substitution effect:

Old bundle is $x_1 = \frac{40}{3}, x_2 = \frac{40}{3}$. The cost of this bundle under the new prices is:

$$2\frac{40}{3} + 2\frac{40}{3} \quad = \quad \frac{160}{3}$$

What does the consumer buy at the new prices with compensated income?

$$2x_1 + 2x_2 = \frac{160}{3}$$

$$x_1 = \frac{m}{p_1 + p_2}, x_2 = \frac{m}{p_1 + p_2}$$

$$x_1 = \frac{\frac{160}{3}}{2+2}, x_2 = \frac{\frac{160}{3}}{2+2}$$

$$x_1 = \frac{40}{3}, x_2 = \frac{40}{3}$$

Difference between old demand for $x_1$ and demand under compensated budget with new prices is 0.

$$\frac{40}{3} - \frac{40}{3} = 0$$

Substitution effect is 0. Total effect is $\frac{10}{3}$. All of this total effect is due to income.

## 1.6 Example Problem (Perfect Substitutes)

Suppose $u = x_1 + x_2$. Suppose $p_1 = 1$, $p_2 = 3$, $m = 40$. $p_1$ changes to 4. Find the substitution and income effect.

Under the original bundles:

$$(40, 0)$$

After the price change:

$$\left(0, \frac{40}{3}\right)$$

How would I need to buy $(40, 0)$ at the new prices? $\tilde{m} = 160$. What would the consumer buy on the budget line $4x_1 + 3x_2 = 160$?

$$\left(0, \frac{160}{3}\right)$$

All of the 40 unit total effect is due to substitution. zero of the effect is due to income.

## 1.7 Example Problem (Perfect Substitutes Redux)

Suppose $u = x_1 + x_2$. Suppose $p_1 = 1$, $p_2 = 3$, $m = 40$. $p_1$ changes to 2. Find the substitution and income effect.

Original Demand:

$$(40, 0)$$

New Demand:

$$(20, 0)$$

How much would I need to buy the old bundle at the new prices? $\tilde{m} = 80$. What would I buy with 80 at the new prices?

$$(40, 0)$$

The substitution effect is zero. The whole of the 20 unit change in demand for $x_1$ is due to the income effect.