Buying and Selling

Up to now the income of a consumer is “exogenous” it’s it is determined outside the model. “The consumer has income m”.

Instead of having income be an *exogenous* amount of money, the consumer has an “endowment” of goods.

$m \rightarrow (\omega_1, \omega_2)$ this is an bundle of goods the consumer starts with.

A apple farmer consumes pies. A pie consists of 2 apples and 1 crust. The apple farmer has grown 10 apples (this is their endwment).

$$u\left(x_1, x_2\right) = min\left\{\frac{1}{2}x_1, x_2\right\}$$

$$(\omega_1, \omega_2) = (10, 0)$$

Market prices are still p_1, p_2 but now the consumer can either buy or sell at these prices.

The budget set is the set of bundles that be acquired through this process of buying and selling.

The budget condition: **the value of the purchased bundle (demand) has to be less than or equal to the value of the endowment.**

The cost of the demanded bundle (**gross demand).**

$$p_1x_1 + p_2x_2 \leq p_1\omega_1 + p_2\omega_2$$

A apple farmer consumes pies. A pie consists of 2 apples and 1 crust. The apple farmer has grown 10 apples (this is their endwment). Suppose the price of apples is 1 and the price of crusts is 1. The farmer’s budget line is:

$$1x_1 + 1x_2 \leq (1 * 10) + (1 * 0)$$

$$x_1 + x_2 = 10$$

Let’s solve for the demand under this budget. The consumer has perfect complements preferences: $u\left(x_1, x_2\right) = min\left\{\frac{1}{2}x_1, x_2\right\}$. The no -waste condition is:

$$\frac{1}{2}x_1 = x_2$$

THe budget equation is:

$$x_1 + x_2 = 10$$

Solve these together:

$$x_1 + \frac{1}{2}x_1 = 10$$

$$\frac{3}{2}x_1 = 10$$

$$x_1 = \frac{20}{3}, x_2 = \frac{10}{3}$$

The gross demand for consumer is $\left(\frac{20}{3}, \frac{10}{3}\right)$.

Gross Demand vs. Net Demand

The consumer's **net demand** is the difference between the **gross demand** and the **endowment.**

$$\left(\frac{20}{3}, \frac{10}{3}\right) - (10, 0) = \left(-\frac{10}{3}, \frac{10}{3}\right)$$

The negative net demand for apples, implies the consumer is a "**net seller**" of apples. The positive net demand for crusts implies the consumer is a "**net buyer**" of crusts.

The budget equation implies if you are a net buyer of one good, you have to be a net seller of the other.

$$p_1 (x_1 - \omega_1) + p_2 (x_2 - \omega_2) = 0$$

Price Changes and Net Buyers/Sellers

The consumer has an endowment of $(3, 3)$. Because the consumer can always just eat the endowment, it is a bundle that is on the budget line regardless of the prices.

This implies when we change the prices, the budget line will "pivot" through the endowment.

To the left of the endowment point, the consumer is a net seller of good 1 and a net buyer of good 2.

To the right of the endowment point, the consumer is a net buyer of good 1 and a net seller of good 2.

Predictions under Price Changes.

If a consumer is a **net buyer of a good** and the **price of that good goes down**, they **have to remain a net buyer** and they are **strictly better off**.

If a consumer is a **net seller of a good** and the **price of that good goes up**, they **have to remain a net seller** and they are **strictly better off**.

Example:

Endowment $(5,5)$. $u(x_1,x_2) = x_1^2 x_2^3$. Suppose the prices are $p_1 = 1, p_2 = 1$.

Construct the budget equation under these prices.

$$1x_1 + 1x_2 = (1*5) + (1*5)$$

$$x_1 + x_2 = 10$$

To solve for the consumer's net demand we need the find the "tangency condition". $u = x_1^2 x_2^3$

$$-\frac{2x_1^1 x_2^3}{3x_1^2 x_2^2} = -\frac{1}{1}$$

$$-\frac{2x_2}{3x_1} = -\frac{1}{1}$$

$$\frac{2}{3}x_2 = x_1$$

$$x_1 + x_2 = 10$$

$$\frac{2}{3}x_2 + x_2 + 10$$

$$\frac{5}{3}x_2 = 10$$

$$x_2 = \frac{\left(\frac{3}{5}\right)10}{1}$$

$$x_1 = 4, x_2 = 6$$

If the price of x_2 went down, this consumer would remain a net buyer of x_2 and be strictly better off.

If the price of x_1 went up, this consumer would remain a net seller of x_1 and be strictly better off.