1 Intertemporal Choice Continued

$$(c_1, c_2)$$

$$(m_1, m_2)$$

Two forms of the budget equation.

Measuring prices in terms of period 1 consumption. Present-value version of the budget equation.

$$c_1 + \frac{1}{1+r} c_2 = m_1 + \frac{1}{1+r} m_2$$

If you want to consumer only in period 1:

$$c_1 = m_1 + \frac{1}{1+r} m_2$$

Present value of income.

Measuring prices in terms of period 2 consumption.

$$(1+r)\, c_1 + c_2 = (1+r)\, m_1 + m_2$$

$$c_1 = 0$$

The y-inercept of the budget equation and we call it the "future value of income"

$$c_2 = (1+r)\, m_1 + m_2$$

$$-\frac{p_1}{p_2} = -\left(1+r\right)$$

1.1 Example Problem

$m_1 = 200$, $m_2 = 600$, and $r = \frac{1}{2}$. Utility is: $u\left(c_1, c_2\right) = c_1 c_2$.

$$c_1 + \frac{1}{1+r} c_2 = 200 + \frac{1}{1+r} 600$$

$$c_1 + \frac{1}{1+0.5} c_2 = 200 + \frac{1}{1+0.5} 600$$

Present-value version of the budget equation.

$$c_1 + \frac{1}{1+0.5}c_2 = 600$$

Future-value version of the budget equation is:

$$(1+0.5)\,c_1 + c_2 = (1+0.5)\,200 + 600$$

$$(1+0.5)\,c_1 + c_2 = 900$$

Future-value of income is 900, the present value is 600.

$$-\frac{c_2}{c_1} = -\,(1.5)$$

$$c_2 = 1.5c_1$$

$$c_1 + \frac{1}{1+0.5}c_2 = 200 + \frac{1}{1+0.5}600$$

$$2c_1 = 600$$

$$c_1 = 300$$

$$c_2 = 450$$

$$(300, 450)$$

Borrower (net buyer of c_1, net seller of c_2)

Lender [saver] (net seller of c_1, net buyer of c_2)

Suppose r decreases to 0.25. Is this consumer a borrow or lender at this new interest rate?

This consumer will remain a borrower and will be strictly better off.

1.2 Comparative Statics

When the interest rate goes down, a borrower remains a borrower and is strictly better off.

When the interest rate goes up, a lender/saver remains a lender/saver and is strictly better off.

2 Market Demand

Aggregate (sum) individual demands to get "market" or "aggregate" demands. Who's demand is it? What good is that demand for?

$$x_{name}^{good}$$

Demand of consumer 2 for good 1 is x_2^1.
The demand for consumer 3 for good 2 is x_3^2.

$$u_1\left(x_1^1, x_1^2\right) = \left(x_1^1\right)^2 \left(x_1^2\right)^1$$

$\left(x_1^1\right)^2$ **is consumer 1's demand for good** 1 **raised to the power of 2.**

$$x_1^1\left(p_1, p_2, m_1\right)$$

$$x_2^1\left(p_1, p_2, m\right)$$

2.1 Summing Demands

If there are n people in the economy. The market or aggregate demand for good 1 and 2 are:
$X^1 = \sum_{i=1}^n x_i^1$.
$X^2 = \sum_{i=1}^n x_i^2$

2.2 Example of Cobb Douglass Demand

Suppose we have 3 consumer's with Cobb Douglass utility. Where x_i^1 is consumer i's demand for good 1 and x_i^2 is consumer $i's$ demand for good 2.

All have utility function: $u_i\left(x_i^1, x_i^2\right) = \left(x_i^1\right)^1 \left(x_i^2\right)^1$ where the $()^1$ indicates a power of 1. All consumers have cobb douglass utility with power 1 on both goods.

Suppose $p_1 = 1$, $p_2 = 1$ and $m_1 = 10$, $m_2 = 20$, $m_3 = 30$.

Find the individual demands for good 1. The demand for consumer i for good 1 is:

$$x_i^1 = \frac{\frac{1}{2}m_i}{p_1}$$

$$x_1^1 = \frac{\frac{1}{2}10}{1} = 5, x_2^1 = \frac{\frac{1}{2}20}{1} = 10, x_3^1 = \frac{\frac{1}{2}30}{1} = 15$$

$$X^1 = 5 + 10 + 15 = 30$$

The market demand for good 1 is 30.

Suppose we didn't know demand:

$$x_i^1 = \frac{\frac{1}{2}m_i}{p_1}$$

Aggregate income: $\sum_{i=1}^{n} m_i = M$

$$\sum_{i=1}^{n} \frac{\frac{1}{2}m_i}{p_1} = \frac{1}{2p_1}\sum_{i=1}^{n} m_i = \frac{1}{2p_1}M$$

Notice that the demand for good 1 only depends on aggregate income and not the distribution of income. **This is called the representative consumer property.** The market acts the same as one consumer who has all the income.

The representative consumer property requires that everyone has the same utility function and that the utility function is **"homothetic"**

2.3 Example of Quasi-Linear Demand

Suppose we have n consumer's Cobb Douglass consumers all have the utility function: $u_i\left(x_i^1, x_i^2\right) = \left(x_i^1\right) + ln\left(x_i^2\right)$. Suppose $p_1 = 1$, $p_2 = 1$ and $m_1 = 10$, $m_2 = 20$, $m_3 = 30$.

Tangency condition:

$$-\frac{1}{\frac{1}{x_i^2}} = -1$$

$$1 = \frac{1}{x_i^2}$$

Tangency condition tells us that the amount of good 2 each consumer chooses is 1.

$$x_i^2 = 1$$

$$X^2 = 3$$

If we gave one of these consumers all of the income $M = 60$ they would choose $x_i^2 = 1$.

This problem does not meet the representative consumer property.

Quasi-linear utility is not homothetic.

2.4 Homothetic Preferences

Preferences are homothetic if $x \succsim y$ implies that $tx \succsim ty$ for any $t \geq 0$.

$$(2,1) \succsim (1,2)$$

$$t = \frac{1}{2} : \left(1, \frac{1}{2}\right) \succsim \left(\frac{1}{2}, 1\right)$$

$$t = 2 : (4,2) \succsim (2,4)$$

The marginal rate of substitution only depends on the ratio of goods but not the absolute value.

Indifference curves are parallel along rays through the origin.

Homothetic:

$$x_1 x_2$$

$$-\frac{x_2}{x_1} = -\frac{tx_2}{tx_1} = -\frac{x_2}{x_1}$$

$x_1 + ln\left(x_2\right)$- Not Homothetic

$$MRS = -\frac{1}{\frac{1}{x_2}} = -x_2$$

$$-x_2 \neq -tx_2$$