

## 0.1 Elasticity

Suppose the price of a good changes from 1 to 2. Demand in market for apples changes from 100 to 50 and

In the market for dragonfruit, the price changes from 1 to 2 and demand changes from 10 to 5.

For apples we have increase of \$1 in price and demand drops by 50.

For dragon fruit we have increase of \$1 in price and demand drops by 5.

For apples we have increase of \$1 in price and demand drops by 50.

For apples we have increase of 1000 yuan demand drops by 50.

$$\frac{\Delta x}{\Delta p} \rightarrow \frac{\% \Delta X}{\% \Delta p}$$

An elasticity measures how something changes in percentage terms when something else changes by some percent.

## 0.2 Elasticities

$$\frac{\frac{100-50}{100}}{\frac{1-2}{1}} = -\frac{1}{2}$$

$$\frac{\frac{\Delta x}{x}}{\frac{\Delta p}{p}} = \frac{\Delta x}{\Delta p} \frac{p}{x} \rightarrow \frac{\partial x}{\partial p} \frac{p}{x}$$

An elasticity measures, for a one percent increase in *something* (price, income) what is the percent change in demand?

Price elasticity of demand:

$$\varepsilon_{i,i} = \frac{\partial x_i}{\partial p_i} \frac{p_i}{x_i}$$

## 0.3 Cobb Douglass Example

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

-1

$$\frac{\partial x_i}{\partial p_i} \frac{p_i}{x_i} = \frac{\partial \left( \frac{\frac{1}{2}m}{p_1} \right)}{\partial p_1} \frac{p_1}{\frac{\frac{1}{2}m}{p_1}} =$$

$$\frac{\partial \left( \frac{1}{2}m p_1^{-1} \right)}{\partial p_1} \frac{p_1}{\frac{\frac{1}{2}m}{p_1}} = -\frac{1}{2}m p_1^{-2} \frac{p_1}{\frac{\frac{1}{2}m}{p_1}} = -\frac{\frac{1}{2}m}{p_1^2} \frac{p_1^2}{\frac{1}{2}m} = -1$$

A 1% percent increase in price leads to a 1% decrease in demand.

Since the absolute value of elasticity is 1 we say the demand is **unit elastic**.

When the absolute value is small  $< 1$  we say demand is **inelastic**. **Demand is insensitive to price.**

Examples of inelastic demand: Medicine. Illegal Drugs. Foods. Gas. Any goods which have few substitutes. Beer at a baseball game.

When the absolute value is large  $> 1$  we say demand is **elastic**. **Demand is sensitive to price.** Goods that have many substitutes. Soft drinks.

## 0.4 Other Elasticities.

Price elasticity for good 1:

$$\varepsilon_{1,1}$$

For an ordinary good, the sign will be negative.

Cross-price elasticity for good 1 with respect to price 2:

$$\varepsilon_{1,2} = \frac{\partial x_1}{\partial p_2} \frac{p_2}{x_1}$$

With complements, the sign will be negative and with substitutes, the demand will be positive.

Calculate the price and cross-price elasticity for perfect complements good.  
 $u = \min \{x_1, x_2\}$

Demand for good 1 is:  $x_1 = \frac{m}{p_1+p_2}$   $x_2 = \frac{m}{p_1+p_2}$

$$\begin{aligned} \varepsilon_{1,1} &= \frac{\partial \left( \frac{m}{p_1+p_2} \right)}{\partial p_1} \frac{p_1}{\frac{m}{p_1+p_2}} = \frac{\partial \left( m (p_1 + p_2)^{-1} \right)}{\partial p_1} \frac{p_1}{\frac{m}{p_1+p_2}} = m (-1) (p_1 + p_2)^{-2} \frac{p_1}{\frac{m}{p_1+p_2}} \\ &= - \frac{m p_1 (p_1 + p_2)}{(p_1 + p_2)^2 m} = - \frac{p_1}{p_1 + p_2} \end{aligned}$$

Since  $p_1 + p_2 > p_1$ , demand is always inelastic.

Suppose  $p_1 = p_2 = 1$

$$\varepsilon_{1,1} = -\frac{1}{2}$$

$$\varepsilon_{1,2} = \frac{\partial \left( \frac{m}{p_1+p_2} \right)}{\partial p_2} \frac{p_2}{\frac{m}{p_1+p_2}} = - \frac{p_2}{p_1 + p_2}$$

$$\varepsilon_{1,2} = 0 \frac{p_2}{\frac{1}{2}m} = 0$$

Income elasticity: “How does demand change in percentage terms when income changes by 1%”

$$\eta_1 = \frac{\partial x_1}{\partial m} \frac{m}{x_1}$$

For cobb douglass. We expect this to be 1.

$$\frac{\partial \left( \frac{\frac{1}{2}m}{p_1} \right)}{\partial m} \frac{m}{\frac{1}{2}m} = \frac{1}{2} \frac{p_1 m}{p_1 \frac{1}{2}m} = 1$$

This is positive since cobb douglass demand is “normal” (not inferior).  
 $u = \min \{x_1, \frac{1}{2}x_2\}$

$$\begin{aligned} \frac{\partial \left( \frac{m}{p_1 + 2p_2} \right)}{\partial m} \frac{m}{\frac{m}{p_1 + 2p_2}} &= \frac{1}{p_1 + 2p_2} \frac{m}{\frac{m}{p_1 + 2p_2}} \\ &= \frac{1}{p_1 + 2p_2} \frac{(p_1 + 2p_2) m}{m} = 1 \end{aligned}$$