1 Technology

Inputs/Outputs

An input is like an ingredient, and output is what the firm sells.
A input/output vector is a vector of amounts of "goods":
Baker uses 2 apples and 1 crust to make a pie.

$$\{(-2,-1,1),(-4,-2,2)\}$$

Two inputs (x_1, x_2), and one output y
x_1 is apples, x_2 is crusts, and y is pies.

1.1 Production Functions

Production function says, given a combination of inputs, what output does it produce?
$f : \mathbb{R}^2_+ \to \mathbb{R}_{:+}$

$$f(x_1, x_2) = y$$

$$f(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$$

$$f(2,1) = min\left\{\frac{1}{2}(2), 1\right\} = min\{1,1\} = 1$$

$$f(4,2) = 2$$

Peach and apple pies.

$$f(2,2,2) = (1,1), f(4,0,2) = (2,0), f(0,4,2) = (0,2)$$

$f(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{1}{3}}$ cobb douglass production
$f(x_1, x_2) = x_1 + x_2$ perfect substitutes production
$f(x_1, x_2) = x_1 + ln(x_2)$ perfect substitutes production
$f(x_1, x_2) = (x_1 + x_2)^{\frac{1}{2}}$ constant elasticity of substitution

Utility functions are "ordinal". The information encoded in the value of the utility function has no "magnitude".

$$u(2,1) = 2, u(1,1) = 1$$

$u(2,1)$ is one more than the utility of $u(1,1)$ is meaningless.

$$2u(2,1) = 4, 2u(1,1) = 2$$

For a production function:

$$f(2,1) = 1, f(4,2) = 2$$

The actual magnitude of the production is meaningful and measured in terms of units of output.

1.2 Properties

Convexity, Monotonicity, Homotheticity. All of these properties apply to production functions in the same way.

Monotonicity as applied to production.

If $x_1 \geq y_1$ and $x_2 \geq y_2$

$$f(x_1, x_2) \geq f(y_1, y_2)$$

$x_1 > y_1$ and $x_2 > y_2$

$$f(x_1, x_2) > f(y_1, y_2)$$

1.3 Isoquants

Consumers an indifference curve is a set of bundles that give the same utiliyt.

An **Isoquant** is a set of input bundles that **produce the same output**.

$$\{(2,1), (2,2), (3,1), (100,1), ...\}$$

Marginal **Technical Rate of Substitution**

The rate at which inputs can be substituted for each-other to produce the same out. Measured by the slope of the isoquant at a point.

How much x_2 can I give up, if I am willing to use one more unit of x_1.

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$$MRS - \frac{\frac{\partial u(x_1,x_2)}{\partial x_1}}{\frac{\partial u(x_1,x_2)}{\partial x_2}}$$

$$TRS = -\frac{\frac{\partial f(x_1,x_2)}{\partial x_1}}{\frac{\partial f(x_1,x_2)}{\partial x_2}}$$

1.4 Marginal Products

Marginal utility $\frac{\partial u(x_1,x_2)}{\partial x_1}$ how much does utility go up when I eat a little more x_1.

Marginal product $\frac{\partial f(x_1,x_2)}{\partial x_1}$ literally measures how much production increases when I increase x_1. *How much extra output do I get if I use one more unit of* x_1.

Example: $f(x_1, x_2) = (x_1 + x_2)^{\frac{1}{2}}$.

1.5 Diminishing Marginal Product

If you increase use of just one of the inputs without increases the others, the extra output you get will get smaller and smaller over time.

$$f(1,1000) = 10, f(2,1000) = 19, f(3,1000) = 27, f(4,1000) = 30$$

$f(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{1}{3}}$

$$MP_1 = \frac{\partial \left(x_1^{\frac{1}{3}} x_2^{\frac{1}{3}} \right)}{\partial x_1} = \frac{1}{3} \frac{x_2^{\frac{1}{3}}}{x_1^{\frac{2}{3}}}$$

Diminishing marginal product because x_1 only appears in the denominator.

$$\frac{\partial \left(\frac{1}{3} \frac{x_2^{\frac{1}{3}}}{x_1^{\frac{2}{3}}} \right)}{\partial x_1} = -\frac{2\sqrt[3]{x_2}}{9x_1^{5/3}}$$

If the derivative of the marginal product is negative, we have diminishing marginal product. That is, if **the second (partial) derivative of the production function is negative**.

$f(x_1, x_2) = x_1^2 x_2^2$

$$\frac{\partial \left(x_1^2 x_2^2 \right)}{\partial x_1} = 2x_1 x_2^2$$

Does not have diminishing marginal product because MP_1 is increasing in x_1

$$\frac{\partial \left(2x_1 x_2^2 \right)}{\partial x_1} = 2x_2^2 > 0$$

1.6 Returns to Scale

How much extra output do I get if I "double" both of the inputs.

Linear returns to scale.

$$f(2,1) = 1, f(4,2) = 2, f(8,4) = 4$$

$$f(2,1) = 1, f(6,3) = 3, f(18,9) = 9$$

For an $t > 0$ $f(tx_1, tx_2) = tf(x_1, x_2)$.

Linear (constant) returns to scale requires: Doubling input, doubles output.

Decreasing returns to scale requires. If I double input, I get less than double the output.

$$f(1,1) = 10, f(2,2) = 15$$

For any $t > 1$, $f(tx_1, tx_2) < tf(x_1, x_2)$.

Increasing returns to scale requires, if I double input, I get more than double the output.

$$f(1,1) = 10, f(2,2) = 30$$

1.7 Example of Cobb Douglass

Suppoese $\alpha > 0$ and $\beta > 0$. When is $x_1^\alpha x_2^\beta$ diminishing marginal product and decreasing returns to scale?

$$f(x_1, x_2) = x_1^\alpha x_2^\beta$$

For what values of α and β does this function have increasing, decreasing or constant returns to scale.

When does this function have diminishing marginal product for x_1?

$$MP_1 = \frac{\partial\left(x_1^\alpha x_2^\beta\right)}{\partial x_1} = \alpha x_1^{\alpha-1} x_2^\beta$$

Let's look at the derivative of MP_1

$$\frac{\partial\left(\alpha x_1^{\alpha-1} x_2^\beta\right)}{\partial x_1} = (\alpha-1)\alpha x_1^{\alpha-2} x_2^\beta$$

When is $(\alpha-1)\alpha x_1^{\alpha-2} x_2^\beta < 0$? Since α, x_1, x_2 are all positive, $\alpha x_1^{\alpha-2} x_2^\beta$ is positive as well. $\alpha - 1$ can be positive or negative depending on whether $\alpha > 1$. Thus, the whole value $(\alpha-1)\alpha x_1^{\alpha-2} x_2^\beta$ is positive if $\alpha > 1$ and negative is $\alpha < 1$. Thus, the function has diminishing marginal product if $\alpha < 1$.