0.1 Bundling

Bundling is possible when a firm sells two different types of things and consumers may have complementary preferences over those things.

	Shirt	Pants	Both
Consumer 1	50	30	80
Consumer 2	10	80	90

zero costs for shirts and pants.
What is the best price to charge for shirts?
If I charge 50 **for a shirt only consumer** 1 **buys and I earn** 50
If I charge 10 for a shirt sell to both and earn 20
If I charge 80 **for pants only consumer** 2 **buys and I earn** 80
If I charge 30 for pants sell to both and earn 60
The best I can do is a total of 130***.***

What if I force the consumers to buy a bundle?
I could charge 90 for a bundle. Only consumer 2 buys and I earn 9
I could charge 80 **for a bundle. Both buy and I earn** 160

0.2 Two-Part Tariff

Consists of two parts. An up-front fee and a "lower" per-unit cost (possibly even free).

Theme park tickets. Pay for the ticket and then rides are free.

Panera month-of-coffee mug. $20 for a mug and then coffee is free.

This only works when consumers demand more than one unit.

A consumer's demand for coffee is $q = 10 - p$ and the firm has zero costs of making coffee.

The inverse demand, $p = 10 - q$.

The firm's profit function for traditional unit pricing:

$$\pi(q) = q(10 - q) - 0$$

$$\frac{\partial(q(10-q))}{\partial q} = 10 - 2q$$

$$10 - 2q = 0$$

$$q = 5, p = 5, \pi = 25$$

But, if I charge $50 for a mug that gets coffee for free. The consumer is just willing to buy it and I earn $\pi = \$50$.

If $mc = 1$ so that $c(q) = q$.

The price per-unit in the two part tariff is $p = 1$. Fee is the consumer's surplus under $p = 1$ which is $\frac{9*9}{2} = 40.5$

1 The Cournot Model of Competition

Monopoly I charge the most I could possibly charge to sell q units.

$$\pi = qp(q) - c(q)$$

Perfect competition, I assume price is fixed

$$\pi = q\bar{p} - c(q)$$

At the optimum for perfect competition

$$p = mc(q)$$

1.1 Cournot Oligopoly

We want to write a model that includes both monopoly and perfect competition as special cases and accounts for things in between.

There are n firms: i is the name of a firm $i \in \{1, 2, ..., n\}$

q_i**: firm** i**'s quantity**

Q: Total (market) quantity $Q = \sum_{i=1}^{n} q_i = q_1 + q_2 + ... + q_n$

Q_{-i}: Total (market) quantity of all firms except i. $Q_{-i} = Q - q_i$.

$$\pi_i = q_i p(Q) - c(q_i)$$

1.2 Example of Maximizing Profit with Two Firms

Suppose inverse demand is $p(Q) = 100 - Q$, there are two firms, and the cost function of each firm is $c(q_i) = 10q_i$.

$$\pi_1 = q_1 (100 - (q_1 + q_2)) - 10q_1$$

$$\pi_2 = q_2 (100 - (q_1 + q_2)) - 10q_2$$

Let's focus on firm 1.

$$\pi_1 = q_1 (100 - (q_1 + q_2)) - 10q_1$$

Suppose firm 1 knew that firm 2 is going to produce 50 units.

$$\pi_1 = q_1 \left(100 - (q_1 + 50)\right) - 10q_1$$

Let's maximize this:

$$\pi_1 = 100q_1 - q_1^2 - 50q_1 - 10q_1$$

$$= 40q_1 - q_1^2$$

$$\frac{\partial \left(40q_1 - q_1^2\right)}{\partial q_1} = 40 - 2q_1$$

$$q_1 = 20$$

The **best response** for firm 1 when firm 2 chooses $q_2 = 50$ is $q_1^* = 20$

$$\pi_2 = q_2 \left(100 - (20 + q_2)\right) - 10q_2$$

$$\frac{\partial \left(q_2 \left(100 - (20 + q_2)\right) - 10q_2\right)}{\partial q_2} = 70 - 2q_2$$

$$35 = q_2$$

Firm 2's best response to firm producing 20 is 35 **not 50**

1.2.1 Game Theory

This model is a Game.

Players, Strategies (actions available), Payoffs.

The players are the firms. The strategies are $q_i \geq 0$. The payoffs are the profit functions.

A solution concept is a way of "solving" a game. Or making predictions about the strategies the players will choose.

Nash equilibrium. Is a set of strategies that are mutual best responses.

q_1 is the best response against q_2 and q_2 is the best response to q_1.

There no incentive to change strategies.

$$(q_1, q_2)$$

$$\pi_1 = q_1 \left(100 - q_1 - q_2\right) - 10q_1$$

$$\frac{\partial\left(q_1\left(100-q_1-q_2\right)-10q_1\right)}{\partial q_1} = -2q_1-q_2+90$$

$$-2q_1-q_2+90=0$$

Best response function for firm 1:

$$\frac{90-q_2}{2}=q_1$$

$$\pi_2=q_2\left(100-q_1-q_2\right)-10q_2$$

$$\frac{\partial\left(q_2\left(100-q_1-q_2\right)-10q_2\right)}{\partial q_2} = -q_1-2q_2+90$$

Best response function for firm 2:

$$\frac{90-q_1}{2}=q_2$$

We need to solve both of these equations simultaneously.

$$\{\{q_1\to 30, q_2\to 30\}\}$$